Otto-von-Guericke-Universität Magdeburg

Fakultät für Informatik
Lehrstuhl für Simulation

1. August 2005

Written Examination (Klausur) *Advanced Discrete Modelling*

Total number of points achievable:	90
Number of questions:	6
Number of pages:	8
Time allowed:	120 minutes
Material allowed:	none

Name:			
Matrikelnummer:		Studiengang/Matrikeljahr:	

You may answer in English or German (or both!)

Ihre Antworten dürfen auf englisch oder auf deutsch (oder beides!) sein.

Zur Information:

Aus den Vorgaben zur Durchführung schriftlicher Prüfungen der Fakultät für Informatik:

Wir machen Sie darauf aufmerksam, dass Täuschungsversuche, z.B. die Benutzung nicht zugelassener Hilfsmittel oder Ordnungsverstöße zur Bewertung der Klausur mit der Note „nicht ausreichend" führen. Sowohl Täuschungsversuche als auch Ordnungsverstöße werden protokolliert. Ordnungsverstöße können nach einer Abmahnung zum Ausschluss von der Klausur führen. Bei Täuschungsversuchen können Sie die Klausur zwar fortsetzen, sie wird aber später mit 5,0 bewertet.

Question	1	2	3	4	5	6
Points						

—— Der Lehrstuhl für Simulation wünscht Ihnen viel Erfolg! ——

Question 1: Modelling with Stochastic Petri Nets (10 points).

We are observing a pedestrian crossing which is controlled by a traffic light (Fußgängerampel).

The pedestrian light can be either red or green. Pedestrians arrive at random intervals I. When the traffic light is green, they immediately cross the street. When the light is red, they either wait for the light to change (with probability p) or leave. The duration of the green phase is described by the random variable G, and the duration of the red phase by R.
Assume that in the beginning there is no pedestrian waiting at the traffic light, and that the traffic light is red.
Draw a stochastic Petri net model of this system.

Question 2: Reachability graph and Markov chain (25 points).

Assume in question 2 that all activities in the model in Question 1 are exponentially distributed.

a) Draw the reachability graph of the SPN in Question 1, indicating any vanishing markings.

b) To make the state space of the model finite, limit the number of people that can wait at the traffic light to one. Draw the modified reachability graph, indicating any vanishing markings.

c) Draw the continuous-time Markov chain derived from this new reachability graph. Assume $p = 0.5$ and the rates 0.5/min for I, 2/min for G, 1/min for R.
(Hint: Use a Markov chain with three states)

d) Write down the equations that describe the continuous behaviour of this Markov chain over time.

e) Assuming that time has been discretized into steps of size Δ=0.1, write down the equations for a discrete-time Markov chain (DTMC) whose behaviour approximates that of this CTMC. Is this a good choice for Δ? Why?

Question 3: Proxels (15 points).

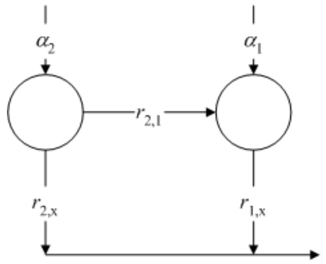
a) Assume that there is no pedestrian waiting at the traffic light, and the traffic light is red at time t=0 in the SPN in Question 1 and the transitions *R* and *G* are not exponentially distributed. Draw the tree of proxels generated by the proxel simulation algorithm for the first three discrete time steps $t = 0, \Delta, 2\Delta$. For one major branch of the tree, include expressions in your diagram that show how the transition probabilities can be computed.

b) Draw the DTMC that corresponds to this proxel computation.

Question 4: Phase-type distributions (10 points).

We now assume that some of the activities in the model in Question 1 (the version with the finite state space) are no longer exponentially distributed. We wish to simulate the model as a Markov chain using continuous phase-type distributions to approximate these non-exponential activities. The distributions of the random variables *R* and *I* are still exponential. Distribution *G* is to be represented by the two-phase approximation shown below.

$G \sim$



Draw the resulting CTMC for the model.

Question 5: Fault Trees (10 points).

a) Consider a car standing at the traffic light:

If there is no air circulation in the car and it gets too warm, then the car overheats. The temperature can rise uncomfortably when the heating is on or the sun is shining. Air can circulate either through an open window, or through the ventilation system.
Name the basic events and construct the fault tree for the above described system. The top event is “overheating of the car”.

b) Perform a qualitative analysis for the above fault tree.

c) Perform a quantitative analysis: The probabilities for the basic events are all 0.5.

d) What property must the basic events fulfill in order to be able to use them in a fault tree?

Question 6: Semester Assignment: Warranty Model (20 points).

a) Draw the Stochastic Petri Net that describes the Vehicle Warranty model from the semester assignment.

b) Draw the tree of proxels generated by the proxel simulation algorithm for the first three discrete time steps $t = 0, \Delta, 2\Delta$. Explain the meaning of the different elements on one of these proxels.

c) What advantages does a simulation based on proxels have over discrete-event simulation?

d) What are the disadvantages?