Department of Knowledge Processing
and Language Engineering
Computational Intelligence
Prof. Dr. R. Kruse, P. Held

Magdeburg, 2012-02-15

Written exam “Bayesian Networks”

Name, first name:	Faculty:	Course:	Matriculation no.:
Type of exam: ☐ First attempt ☐ Second attempt ☐ Certificate	Signature invigilator:		#Sheets:

Task 1	Task 2	Task 3	Task 4	Task 5	Task 6	Sum
/15	/8	/9	/11	/10	/8	/61

Task 1 Bayesian Theorem (7 + 4 + 4 = 15 20 min)

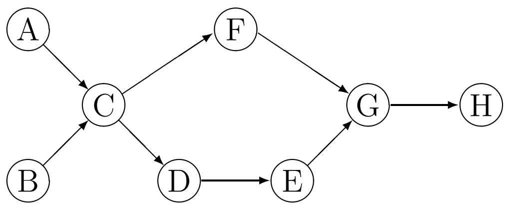
a) In a given population, 5% of all persons suffer a certain desease. Let a test have the property that it correctly recognizes an ill person with 90% probability whereas the rate of correctly revealing a healthy person in 95%. What is the probability that a person does (not) suffer from the desease if the test does (not) reveal the desease?

b) Consider two urns. Urn 1 contains two white and one red ball, urn 2 one white and two red. First, a ball from urn 1 is randomly chosen and placed into urn 2. Finally, a ball from urn 2 is picked. This ball be red: What is the probability that the ball transferred from urn 1 to urn 2 was white?

c) Imagine the roads in winter. With a probability of 30 % the road is slippery. On slippery roads exists a risk of traffic jam caused by accidents of 70 %, while the risk on clear roads is only 20 %. What is the probability that the road was slippery when Prof. Kruse caught up in a traffic jam?

Task 2 Separation Criteria (4 + 4 = 8 15 min)

Consider the following directed graph:



i) $F \perp\!\!\!\perp H \mid G$

ii) $C \perp\!\!\!\perp G \mid F$

iii) $F \perp\!\!\!\perp E \mid C$

iv) $A \perp\!\!\!\perp B \mid \emptyset$

v) $A \perp\!\!\!\perp B \mid D$

vi) $D \perp\!\!\!\perp F \mid \{C, G\}$

vii) $E \perp\!\!\!\perp F \mid \{A, B\}$

viii) $C \perp\!\!\!\perp E \mid \{D, F, H\}$

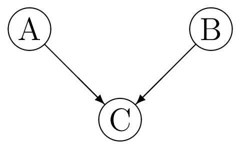
a) Which of the following propositions hold true in the graph??
(„$X \perp\!\!\!\perp Y \mid Z$“ denotes „$X$ and Y are d-separated (in G) by Z.“)

b) Consider the undirected graph that is obtained if all arrow heads from the directed graph are dropped. Check again the propositions i)–viii), now with the u-separation criterion! Which differences can be observed?

Task 3 Bayesian Networks (9 15 min)

Consider the following three-dimensional probability distribution:

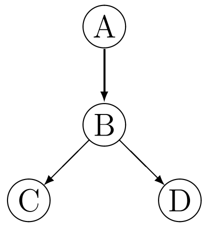
p_{ABC}	$A = a_1$		$A = a_2$	
	$B = b_1$	$B = b_2$	$B = b_1$	$B = b_2$
$C = c_1$	$^3/_{32}$	$^1/_4$	$^1/_{40}$	$^3/_{32}$
$C = c_2$	$^1/_{32}$	$^1/_8$	$^1/_{10}$	$^9/_{32}$



Check whether the graph depicted next to the table can be the underlying network structure describing the distribution! If yes, specify the probability distributions that are needed to define the Bayesian network!

Task 4 Probabilistic Propagation (4 + 7 = 11 30 min)

Consider the following Bayesian network and the corresponding (conditional) probability distributions:



$P(A)$	a_1	a_2
	0.4	0.6

$P(B\|A)$	a_1	a_2
b_1	0.2	0.7
b_2	0.8	0.3

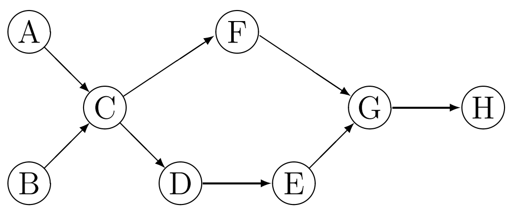
$P(C\|B)$	b_1	b_2
c_1	0.4	0.9
c_2	0.6	0.1

$P(D\|B)$	b_1	b_2
d_1	0.6	0.3
d_2	0.4	0.7

a) Determine the a-priori distribution for all four variables!

b) It becomes evident that variable C assumes value c_2. Propagate this evidence across the network with the tree-based propagation algorithm presented in the lecture, i.e., compute all four a-posteriori distributions!

Task 5 Construction of Clique Trees (3 + 2 + 5 = 10 10 min)



Construct stepwise for the depicted Bayesian network

a) the moral graph,
b) a triangulated moral graph, and
c) a cliquen tree/join tree!

At which steps of the construction do you have multiple options to proceed?

Task 6 Learning from Data (8 30 min)

Assume the following conditional independencies between the attributes A, B, C, D, E, F, G and H (as in former exercises, the notation $X \perp\!\!\!\perp Y \mid Z$ states that X is independent of Y given Z):

$A \perp\!\!\!\perp B \mid \emptyset$	$AB \perp\!\!\!\perp DEFGH \mid C$	$GH \perp\!\!\!\perp F \mid C$
$ABCFGH \perp\!\!\!\perp E \mid D$	$ABC \perp\!\!\!\perp H \mid G$	$H \perp\!\!\!\perp F \mid G$
$ABCG \perp\!\!\!\perp DE \mid FH$	$ABC \perp\!\!\!\perp DE \mid FG$	$G \perp\!\!\!\perp DE \mid CH$

Assume further that only these independencies as well as those that are deducible by the graphoid axioms (cf. lecture slides) hold true (i.e. the symmetric conuterparts $B \perp\!\!\!\perp A \mid \emptyset$ etc. are satisfied). All other conditional independencies do not hold true. Which conditional independence graph over the four attributes can be read from this information?
(Hint: Remember the special properties of converging edges.)