July 19, 2011

Exam *Introduction to Simulation/Modelling and Simulation*

(underline the course that you will take the exam in and strike out the other)

Total number of points:	90
Number of questions:	8
Number of pages:	12 (including appendix and empty pages)
Time limit:	120 minutes
Additional material allowed:	Dictionary

Name:			
Student ID#:		Course of studies/ year	

For your information:

Die Antworten können auch in deutscher Sprache erfolgen.
You may answer the questions either in German or in English.

Rules for written exams at the "Fakultät für Informatik":

Cheating, attempted cheating (e.g. usage of prohibited additional material, copying from other students, etc.) and unruly behavior will result in a "failed" grade for the exam. Any violation of the rules will be recorded. In the case of cheating or attempted cheating the student may choose to continue the exam even though it will be graded as "failed". In case of unruly behavior, students will be warned once, and in case of recurrence will not be allowed to finish the exam.

Note: The courses "Introduction and Simulation" and "Modelling and Simulation" have identical content. Students may therefore take an exam in at most one of the two courses!

Question	Points	
1		
2		
3		
4		
5		
6		
7		
8		
Total:		

—— The simulation group wishes you good luck! ——

Questions 1: Continuous modeling [10 Points]. Global Warming.

Global warming is a recurring hot topic. Many influences exist, but our model will be limited to a few of them. The model is purely fictional and heavily simplified! It considers only five *positive*, *real-valued* influences:

- Temperature of the atmosphere: A
- Earth surface temperature: E
- Amount of greenhouse gases in the atmosphere: G
- World Population: P
- Number of trees on the planetary surface: W

The following assumptions are made regarding the interdependencies of these population groups:

- The temperature of the atmosphere changes proportionally to its difference to the surface temperature, so that both equalize in the long run.
- At the same time, it declines proportionally to itself, since heat energy is emitted into outer space.
- Likewise, the surface temperature changes proportionally to its difference to the temperature of the atmosphere
- Furthermore, the temperature of the atmosphere increases by a value that is proportional to both, the solar radiation S and the amount of greenhouse gases in the atmosphere.
- The surface temperature declines proportionally to the number of trees in the planetary surface, since forests cool the air due to the evaporation of water.
- The amount of greenhouse gases increases proportionally to the current world population. However, it also declines proportionally to the number of trees, since the trees are able to absorb those gases.
- The growth of the world population follows the *logistic equation* with a maximum of $P_{Max.}$
- The number of trees declines proportionally to the world population.
- However, at the same time, the forests regenerate and increase in size proportionally to their current size, and inversely proportional to the absolute difference between the current surface temperature and the optimal temperature T_{opt}.

a) [9 Points]

Describe this model *as a system of ordinary differential equations*! Use symbols a_1, a_2, etc. for positive constants.

b) [1 Point]

Mark the one of the following questions that can**not** be answered by such a model!

1. What is the probability that all trees will be gone at some point in the future?
2. How does the amount of greenhouse gases change in the long term?
3. Are we headed for a global warming?

Question 2: Semester Assignment „Star Wars Episode 6: Return of the Jedi" [20 Points].

a) Continuous/Hybrid Behavior [10 Points]
Sketch a typical development of „the death star capacitor charge level"! Briefly **explain** the behavior! **Mark** and **name** at least four (in total!) different activities, states and events!

b) AnyLogic-Modeling [5 Points]
Explain in short, how and with the use of which AnyLogic model elements you modeled the following:
„an Ewok attacks the final imperial walker" (also consider the consequences of this state!)

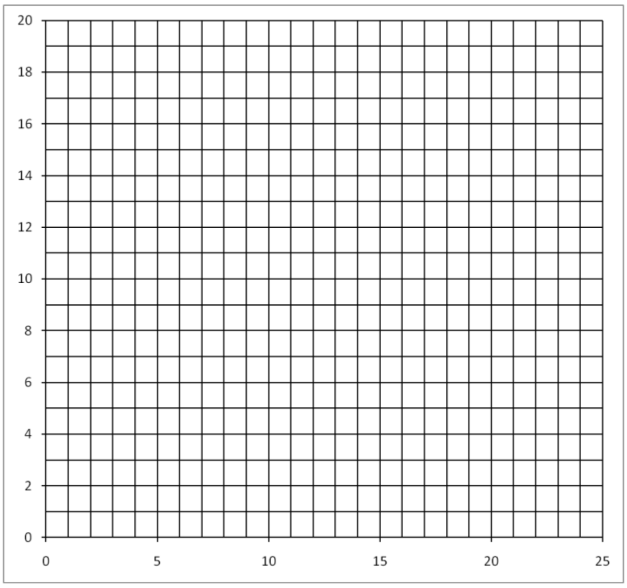
c) The Death Star [5 Points]
How often will the death star fire its main weapon during the space battle? **Give** a statistically meaningful answer! **Explain**, what it means! **Describe**, on what basis it has been obtained!

Question 3: Input data analysis [10 Points].

a) Quantile-Quantile-Plot [5 Points]
The following ten numbers were obtained in a measurement:

1.9   19.5   13.5   0.3   4.6   0.7   9.5   7.0   3.1   5.4

You assume that these measurements are distributed according to an exponential distribution. To check this assumption, **draw** a Quantile-Quantile-Plot in the empty graph below **and interpret** the result!



b) Chi-Square-Test [5 Points]
Assume that you have received a file containing 100 numbers between 0 and 1. These are assigned to ten intervals ("Observed") according to their value. Judging from the histogram of these intervals, someone claims that the numbers are uniformly distributed in the interval [0, 1].

	xMin	***xMax***	***Expected***	***Observed***		
	0	0.1		4		
	0.1	0.2		8		
	0.2	0.3		13		
	0.3	0.4		12		
	0.4	0.5		15		
	0.5	0.6		11		
	0.6	0.7		7		
	0.7	0.8		11		
	0.8	0.9		14		
	0.9	1.0		5		
Sum				100		

Perform a Chi-squared test to check that guess! Do not merge any classes. Round numbers to one decimal place. Use first $\alpha = 0.1$ and then $\alpha = 0.05$. **Interpret** the results for both cases!

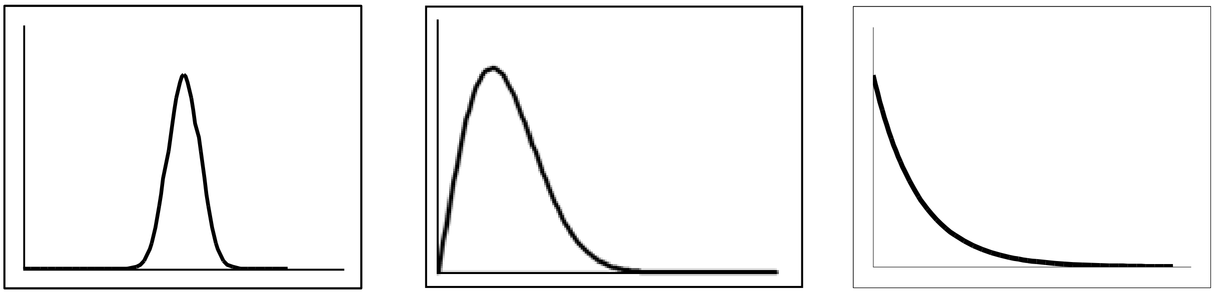
Question 4: Random Variables [10 Points]. Production of Glass Bottles.

a) Probability Density Functions [6 Points]

In conjunction with the production of returnable glass bottles, the company „RenewABottle“ measured the following random variables:

1. The life expectancy of a fork lift that is used on a daily basis
2. The inter-arrival times of trucks loaded with quartz sand at the production facility
3. The duration of the transport of empty bottles with a fork lift from storage to the freight train

The probability density functions of these distributions are shown here:



A B C

Match the graphs A, B and C to the measurements 1, 2 and 3 and **justify** your decision! (Note: Simply naming the corresponding distribution functions or using the process of elimination is not a valid justification.)

b) Distribution Functions [4 Points]

The thickness (in mm) of the bottom of glass bottles produced at „RenewABottle“ is N(30;4) distributed. **Determine**, how many of the 5000 bottles produced on a given day are inside the allowed margin between 27.5mm and 32.5mm!

Question 5: Petri Nets [10 Points]. The Cycle of Returnable Bottles.

Returnable bottles are filled and emptied multiple times over the course of their life span. The following descriptions presents this cycle in an abstract way:

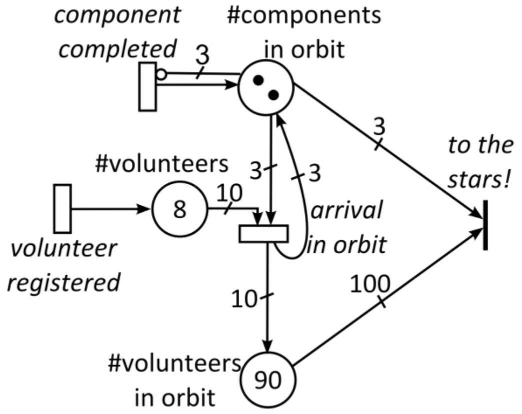
Bottles are returned to the soft drink factory in normally-distributed time intervals and are then being collected in the corresponding buffer in front of the bottling line (the part of the facility that fills the beverage into the bottles). Furthermore, there is also a limited supply of empty crates in front of the bottling line. As soon as there are 20 empty bottles and a crate available, the bottles are instantaneously put into the crate and the filled create enters the bottling line.
The duration of filling all empty bottles of the crate is constant. However, the machine used to fill the bottles fails in exponentially-distributed time intervals. The duration of a failure is Weibull-distributed. After the failed machine has been repaired, the interrupted filling of the bottles in the current crate continues.
Once the bottling is finished, the full bottles are instantaneously removed from the crate and are sold in the factory outlet store. The crate itself is returned to the buffer in front of the bottling line.
A fraction p of the bottles in the outlet store are breakage and have to be discarded, the remaining bottles are sold. Customers buy those one at a time in normally-distributed time intervals. The sold bottles are then emptied and returned by the customers to the buffer in front of the bottling line in uniformly-distributed time intervals.

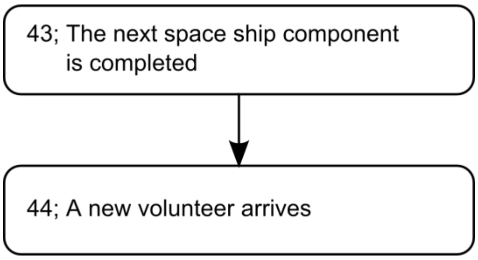
Draw a Petri net model of this system! Assume the following initial state: There are currently two bottles and one crate in the buffer in front of the bottling line and one crate is currently being refilled. There is one bottle at the customers. **Mark** all transitions that have a race age policy! **List** the transitions that are currently enabled!

Question 6: Progression of a discrete simulation [10 Points]. The colonization of outer space.

The construction of human colonies in outer space has become widespread. Volunteers may register at any time at the recruitment center on earth. The space ships consist of three components, which are built one after another, are lifted into Earth' orbit and are being assembled there. Once all three components are in orbit, the construction of new components is interrupted. As soon as the required crew of 100 people has registered, the space ship sets off to its destination and the construction of the next space ship starts.

The following Petri net represents this system: At time 42, two space ship components are already assembled in orbit. Eight volunteers have registered at the recruitment office on Earth and 90 volunteers are already in orbit. The *Future-Event-List* (FEL) of the system is the following:





The next three time intervals for the production of space ship components are: 4, 7 and 8.
The next three time intervals between registrations of volunteers are: 1, 1 and 3.
The next three durations of volunteer transports into orbit are: 2, 1 and 2.

a) Simulation progression [7 Points]
Sketch the progression of the simulation program from time 42 to time 47. **Present** the system state after each state change and **state** the event that caused it. **Mark**, which events are primary and which secondary!

b) Future Event List [3 Points]
Describe or draw the FEL for time 47, i.e. the FEL after events for that point in time have been processed!

Question 7: Output-Analysis [10 Points]. A Multiplex Cinema

The operating company of a number of multiplex cinemas wants to build a new cinema, but has yet to decide on a configuration. Configuration 1 has few big screens, configuration 2 has numerous smaller ones. It is now your task to determine, which configuration is more profitable.

Fortunately, profits figures of two cinemas that correspond to the two possible configurations are available. For both, the profits of a ten month period have been recorded:

Month	Configuration 1	Configuration 2	D_r	$\overline{D}$	$(D_r-\overline{D})^2$		
1	39	39					
2	64	68					
3	40	43					
4	52	53					
5	37	43					
6	59	63					
7	54	61					
8	45	45					
9	51	49					
10	50	57					

a) Comparison [10 Points]
Which configuration should be built? Interpret the results of your calculations and justify your decision. (Hints: use empty table cells for your calculations. For the computation of square roots rough estimates are sufficient)

Question 8: Miscellaneous [10 Points].

a) Given the *initial value problem* $y' = 2y - 4t$, $y(0) = 2$. Using the Forward Euler method with step size $\Delta t=1$, **compute** the value of y at time $t = 3$! [3 Points]

b) **Generate** (pseudo) random numbers that are **N(30;4)** (!) distributed using the *Linear Congruential Method!* **Compute** the first four values generated with this method using the parameters $a = 3$, $c = 5$, $m = 100$ and the seed $x_0 = 12$! [3 Points]

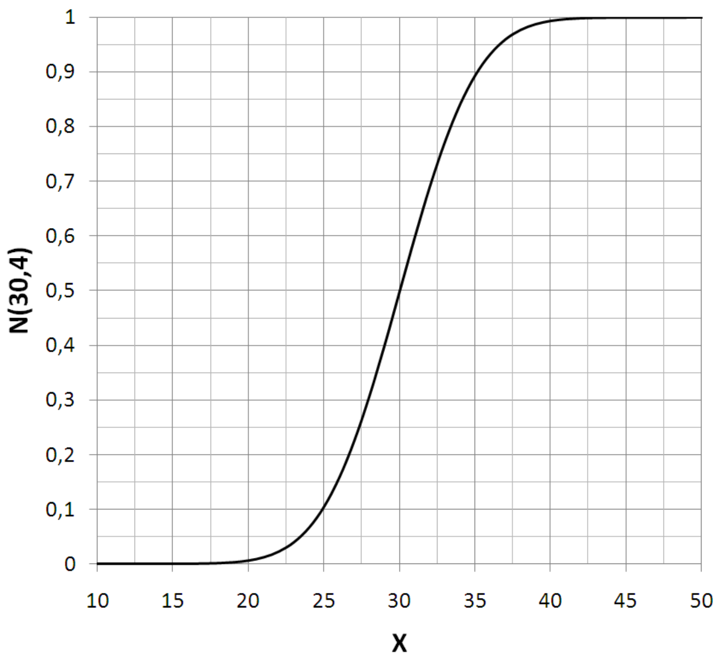
c) We are considering the university library. **Give** an example for each of the following (according to the definitions given in the lecture!) [3 Points]

- an event –
- an activity –
- a delay –
- an entity –
- an attribute –
- a state variable –

d) A queue has formed in front of the examination office. Students arrive there about every five minutes and the queue holds on average three people. How long should a student expect to have to wait in the queue? [1 Point]

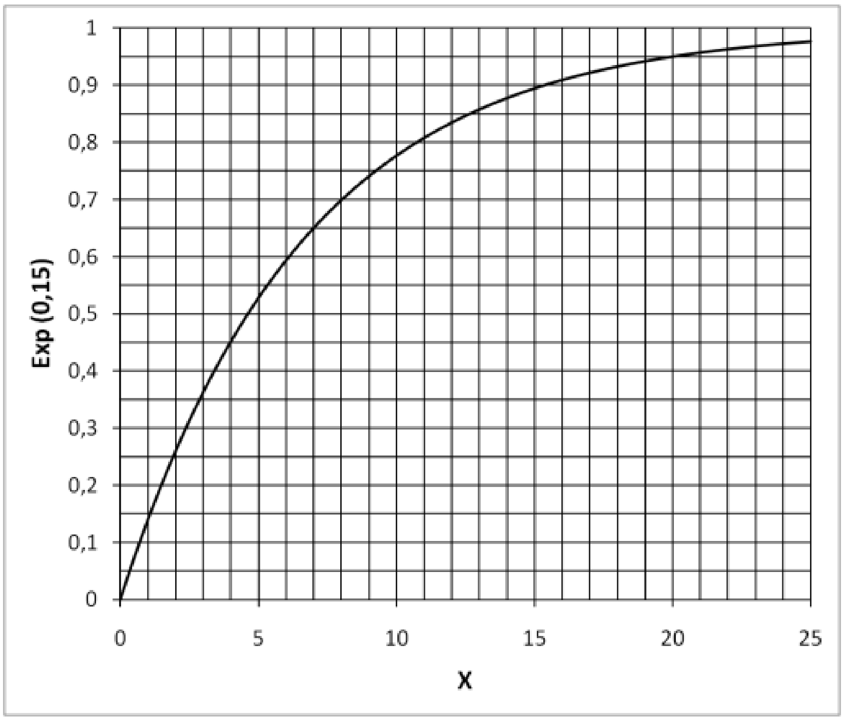
Appendix

Graph of the N(30;4) Distribution



The value of the Student *t*-distribution for 9 degrees of freedom at position 0.05 is 2.26

Graph of the Exp(0.15) Distribution



Some values of the χ^2-Distribution:

		#degrees of freedom					
		7	8	9	10	11	12
α	0.05	14.07	15.51	16.92	18.31	19.68	21.03
	0.10	12.02	13.36	14.68	15.99	17.28	18.55