OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG | INF | FAKULTÄT FÜR INFORMATIK

Simulation Research Group

July 24th, 2012

Exam *Introduction to Simulation*

Total number of points obtainable:	100
Number of questions:	9
Number of pages:	13 (including appendix and empty pages)
Time limit:	120 minutes
Additional material allowed:	Dictionary

Name:			
Student ID#:		Course of studies, year of matriculation	

For your information:

You may answer the questions in either German or English.
Answer all questions according to the contents taught in the lecture.

Rules for written exams at the "Fakultät für Informatik":

Cheating, attempted cheating (e.g. usage of prohibited additional material, copying from other students, etc.) and unruly behavior will result in a "failed" grade for the exam. Any violation of the rules will be recorded. In the case of cheating or attempted cheating the student may choose to continue the exam even though it will be graded as "failed". In case of unruly behavior, students will be warned once, and in case of recurrence will not be allowed to finish the exam.

Question	Points	
1		
2		
3		
4		
5		
6		
7		
8		
9		
Total:		

—— The simulation group wishes you good luck! ——

Questions 1: Continuous modeling [10 Points]. In the kids' room.

John and his younger sister Sookie are playing with some ants that somehow found their way into the kids' room. Their parents, however, disapprove.
This model considers the following four positive real-valued quantities:

- The mood of John — J
- The mood of Sookie — S
- The mood of their parents — P
- The number of ants in the room — A

The system is governed by the following interdependencies of these quantities:

- John loves to play with ants. His mood increases proportionally to the number of ants in the room.
- John is currently at odds with his parents. Therefore, his mood also increases inversely proportionally to the mood of his parents.
- Sookie is very fond of her older brother. Her mood changes proportionally to the difference between his mood and hers, so that both equalize in the long term.
- The number of ants in the room grows according to the logistic equation with maximum A_{max}.
- Since the parents are happy whenever their kids are happy, their mood increases proportionally to the product of the moods of the two children.
- However, since the parents also like their home to be tidy, their mood additionally decreases proportionally to the square of the number of ants in the kid's room.
- Since the parents will clean up the room the worse their mood becomes, the number of ants decreases inversely proportional to the parents' mood.

a) [9 Points]
Describe this model as a system of ordinary differential equations. Use symbols a_1, a_2, etc. for **positive** constants.

b) [1 Point]
Mark the one of the following that continuous models cannot account for.

1. Stiff governing equations
2. Exponential growth
3. Random influences

Question 2: Semester Assignment „Ocean's Eleven" [20 Points].

a) Continuous/Hybrid Behavior [10 Points]
Sketch (graphically) a typical development of „Terry Benedicts Suspicion". Briefly **explain why** the system behaves the way you sketched it. In your graph, **mark** and **name** at least four (in total!) different activities and states.

b) AnyLogic-Modeling [5 Points]
Explain in short how and with the use of which AnyLogic model elements you modeled the following:
„ Terry Benedicts Suspicion" (also consider effects of this part of the model on other events.)

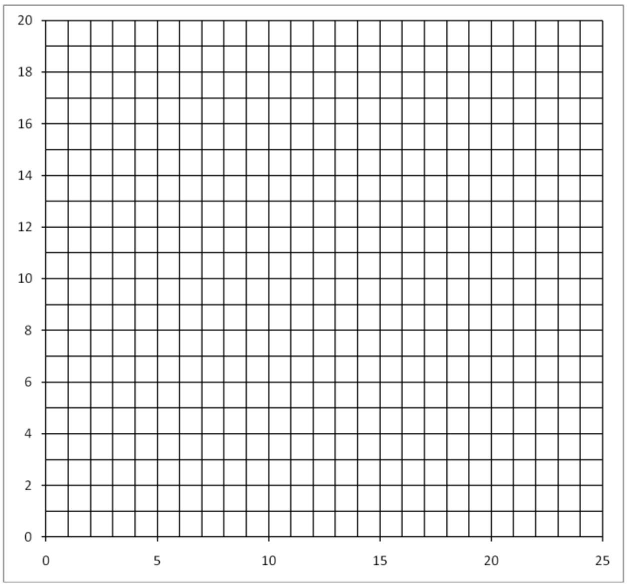
c) Interrupts of Terry Benedict [5 Points]
In the Semester Assignment, your task was to determine how often Terry's suspicion decreases due to interrupts. **Give** a statistically meaningful answer to this task. **Explain** what that answer means. **Describe** on what basis it has been obtained.

Question 3: Input data analysis [10 Points].

a) Quantile-Quantile-Plot [5 Points]
The following ten numbers were obtained in a measurement:

6.9, 9.0, 10.4, 6.1, 16.8, 4.4, 12.4, 5.3, 7.9, 3.2

You assume that these measurements are distributed according to a Lognormal(2, 0.5) distribution. To check this assumption, **draw** a Quantile-Quantile-Plot in the empty graph below **and interpret** the result.



b) Chi-Square-Test [5 Points]
You receive a file containing 100 numbers between 0 and 1. These are assigned to ten intervals ("Observed") according to their value. Someone claims that these numbers are uniformly distributed between 0 and 1.

	xMin	*xMax*	*Expected*	*Observed*		
	0	0.1		11		
	0.1	0.2		12		
	0.2	0.3		12		
	0.3	0.4		8		
	0.4	0.5		5		
	0.5	0.6		14		
	0.6	0.7		7		
	0.7	0.8		12		
	0.8	0.9		15		
	0.9	1.0		4		
Sum				100		

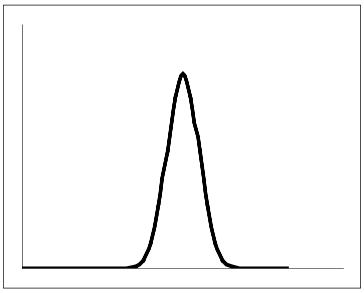
What does the Chi-Square-Test say to this hypothesis? Do not merge any classes; round numbers to one decimal place. Use first $\alpha = 0.1$ and then $\alpha = 0.05$. What exactly do these results mean?

Question 4: Random Variables [10 Points]. At a government agency.

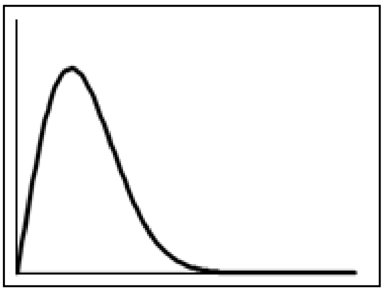
a) Probability Density Functions [6 Points]
At a government office, the “committee to improve efficiency” measured the following random variables:

1. The lifetime of office staplers (devices that bind together pieces of paper)
2. The time intervals between the arrivals of citizens
3. The duration of handling a citizen’s request

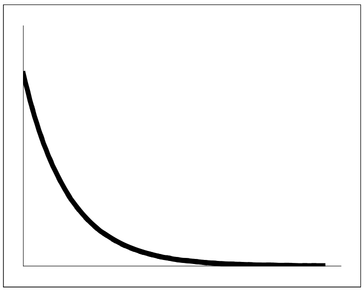
The probability density functions of these distributions are shown here:



A



B



C

Match the graphs A, B and C to the measurements 1, 2 and 3 and **explain** your decision. (Note: Simply naming the corresponding distribution functions is not an explanation for the chosen assignment. The process of elimination is not an explanation either.)

b) Distribution Functions [4 Points]
The duration of lunch breaks at the government agency is normally distributed with mean 30 and standard deviation 4. How many of the 500 lunch breaks made on a given day are inside the legally mandated range between 30 and 35 minutes?

Question 5: Petri nets [10 Points]. A Pit Stop.

Every Formula 1 racing team has two cars running on the track, as long as everything goes well. The pit stop of a car during a race is a well practiced and highly organized routine, where everyone knows their duties. The following description shows the course of events during a pit stop of one racing team while a race is going on. We assume that during each pit stop, all four tires are changed, the car is refueled, and there will be no repairs.

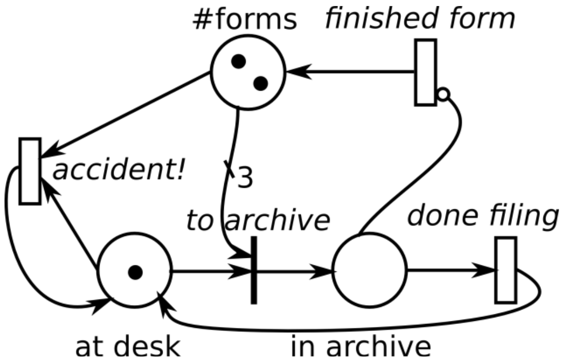
Usually, both cars of the team are on the track. The pit crew, which consists of six engineers is on standby, while there is no car in the pit. Every once in a while one of the engineers needs to use the bathroom, and is absent for a short period of time. During a pit stop, the engineers do not use the bathroom.
After a randomly distributed amount of time, one of the cars enters the pit lane and advances to the pit, which takes a normally distributed amount of time. In order for the actual pit stop to start, all six engineers needs to be on standby. One holds the teams pit sign, four are needed to change the tires and one is refueling the car. Refueling the car and changing the tires take different uniformly distributed time periods.
The fuel pump is a high tech piece of equipment and liable to fail after an exponentially distributed amount of time. These failures are only very short, however, the fuelling is interrupted when the pump fails, and is resumed when the pump is back on line.
When either one of the processes is over, the engineers are again on standby. Only when both the tires and the fueling is done, can the sixth man lift the lollipop and the race car can leave the pit. Leaving the pit lane takes again a normally distributed amount of time.

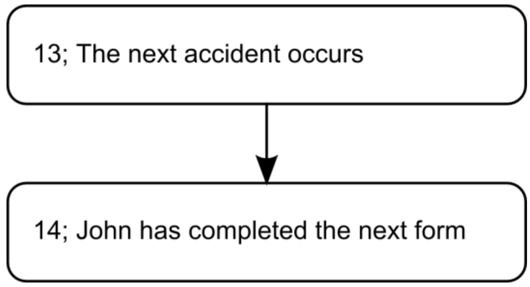
Draw a Petri net model of this system. Assume the following initial state: One of the cars is currently on the track. The pit stop of the other car is under way. The fueling is done, but the tires are still being replaced. Currently the fueling machinery is running.
Mark all transitions that have a race age policy. **List** the transitions that are currently enabled.

Question 6: Progression of a discrete simulation [10 Points]. In a bureaucracy.

John works at a government office. His only job is to fill out and file copies of the Form 1337. He sits at his desk and fills out copies of this form until he has completed three copies. He then takes the forms to the office's archives to file them. Once he is done in the archive, he returns to his desk to fill out more forms. Since this job is very boring, accidents happen whenever John sits at his desk for too long. Each accident destroys one of the filled-out forms.

The following Petri net represents this system: At time 12, two forms have been completed and John is currently at his desk filling out more forms. The *Future-Event-List* (FEL) of the system is the following:





The next three time intervals for filling out forms are: 1, 2, 2
The next three time intervals between accidents are: 6, 4, 3
The next three time intervals for archiving forms are: 2, 1, 1

a) Simulation progression [7 Points]
In order to sketch the progression of the simulation program from time 12 to time 17, **show** the system state and the next event to occur after *each* state change. **Mark** which events are primary and which are secondary (=conditional).

b) Future Event List [3 Points]
Describe or draw the FEL for time 17, i.e. the FEL after all events for that point in time have been processed.

Question 7: Output-Analysis [10 Points]. A Formula 1 Racing Team

The Formula 1 Team Magdeburg Racing needs to select a new type of tire for the next racing season. There are two manufacturers to choose from next season, and the performance of the different tires varies. It is now your task to determine which tire manufacturer the team should select for the next season in order to be competitive.

The team was able to drive ten laps with tires from each of the manufacturers. The only criterion is the speed of the cars, since the wear characteristics of the tires from both manufacturers seem to be identical.
The time in seconds needed for each lap was recorded and can be found in the following table:

Lap	Manufacturer 1	Manufacturer 2	D_r	$\overline{D}$	$(D_r-\overline{D})^2$		
1	75	75					
2	67	73					
3	70	73					
4	80	87					
5	69	69					
6	82	83					
7	84	91					
8	81	79					
9	89	93					
10	94	98					

a) Comparison [10 Points]
Statistically **compare** the two test series. Which tire manufacturer should be chosen? **Interpret** the results of your calculations and **justify** your decision. (Hints: use empty table cells for your calculations. For the computation of square roots rough estimates are sufficient)

Question 8: Discrete Time Markov Chains [10 Points]. The Economy

We want to build a simple model of the economy. We therefore assume, that there are only three main market conditions.

- bull market (increasing investor confidence)
- bear market (transition from high investor optimism to widespread investor fear and pessimism)
- recession (widespread investor fear and pessimism)

We further assume that in any given week, one of these states is prevalent. As the market is very volatile, the situation in the current week only depends on the market situation prevalent last week. Therefore we can assume the dynamics of the economy can be represented by a discrete-time Markov chain (DTMC).

We have data for the 50 weeks from last year, how often one of the market situations was succeeded by any of the other. (e.g. how often a bull market was followed by a recession).

market in week n	market in week n+1	number		
bull market	bull market	15		
bull market	bear market	10		
bull market	recession	25		
bear market	bull market	30		
bear market	bear market	5		
bear market	recession	15		
recession	bull market	5		
recession	bear market	35		
recession	recession	10		

a) Modelling [4 Points]

Sketch the discrete-time Markov chain (DTMC) that can be deduced from this statistic.

b) Transient Solution [4 Points]

Assume that the current week is a bull week. Using the above model, **compute** the probability that two weeks from now we will also have a bull market.

c) *Hidden Markov Models* [2 Points]

Assume that the market situation has a direct influence on the type of car having the highest salesin a given week. The probability that sports cars are sold most is 0.5 in a bull market week, 0.3 in a bear market week and 0.1 in a recession week.

Assuming that the current week is a recession week, **compute** the probability that in the following week, sports cars will have the highest sales.

Question 9: Miscellaneous [10 Points].

a) Given the *initial value problem* y'(t) = 2 y – 3t with y(0) = 1. This problem is to be solved using the Euler method with step size 1. **Compute** the result at time $t = 3$. [3 Points]

b) **Generate** (pseudo) random numbers that are Lognormal(2; 0,5) (!) distributed using the *Linear Congruential Method.* What are the first four values x_1 to x_4 generated using the parameters a=8, c=5, m=100 and the seed x_0=26?
[3 Points]

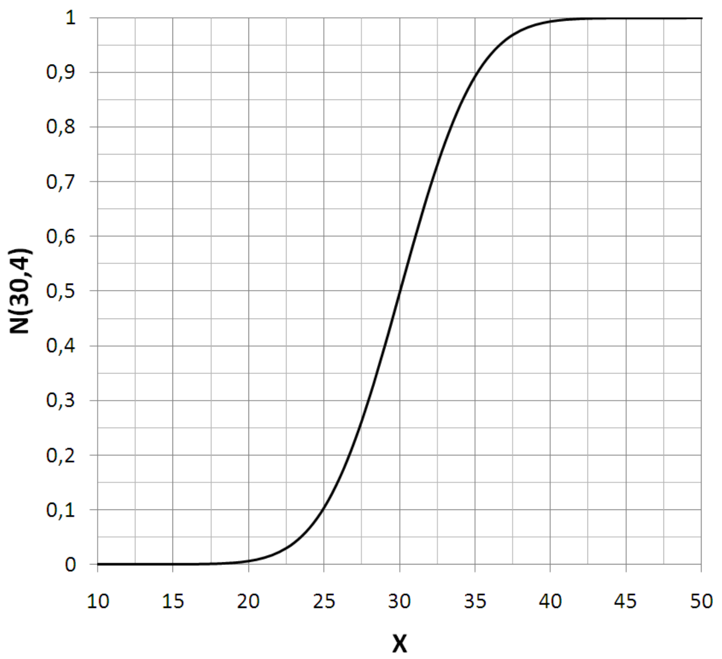
c) We are considering a Formula 1 car race. **Give** an example for each of the following [3 Points]
(Refer to the definitions from the lecture!)

- an event –
- an activity –
- a delay –
- an entity –
- an attribute –
- a state variable –

d) A queue has formed in front of a cold beverage vending machine. People arrive there about every 2 minutes and the queue holds on average 12 people. **Compute** how long a person can expect to wait in the queue on average. [1 Point]

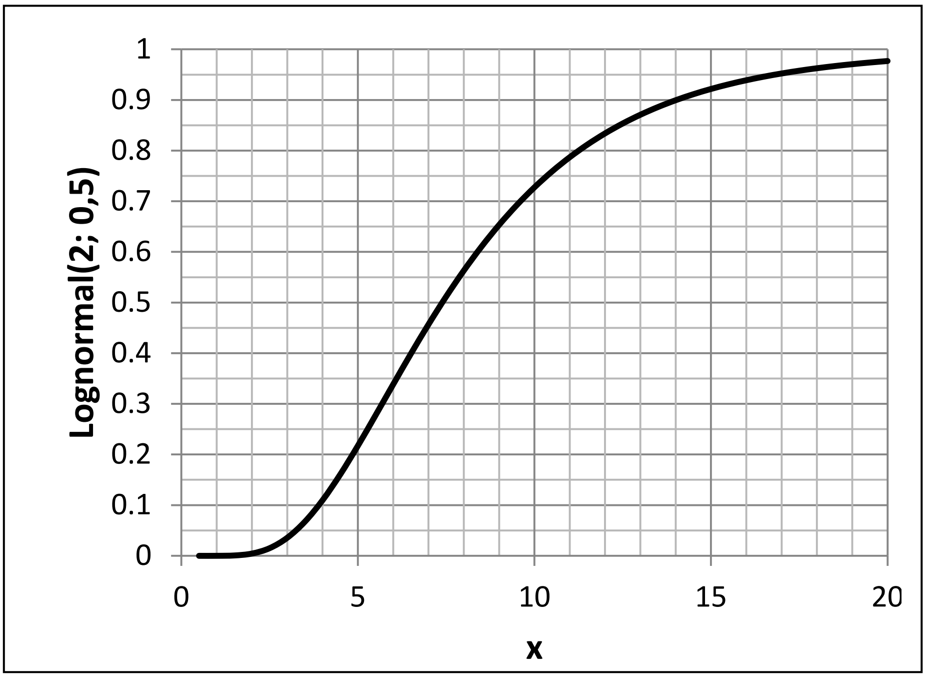
Appendix

Graph of the N(30;4) Distribution



The value of the Student *t*-distribution for 9 degrees of freedom at position 0.05 is 2.26

Graph of the Lognormal(2; 0,5) cumulative distribution function



Some values of the χ^2-Distribution:

		#degrees of freedom					
		7	8	9	10	11	12
α	0.05	14.06	15.51	16.92	18.31	19.68	21.03
	0.10	12.01	13.36	14.68	15.99	17.28	18.55