OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

Simulation Research Group

January 28th, 2013

Exam *Introduction to Simulation*

Total number of points obtainable:	100
Number of questions:	9
Number of pages:	14 (including appendix and empty pages)
Time limit:	120 minutes
Additional material allowed:	Dictionary

Name:			
Student ID#:		Course of studies, year of matriculation	

For your information:

You may answer the questions in either German or English.
Answer all questions according to the contents taught in the lecture.

Rules for written exams at the “Fakultät für Informatik”:

Cheating, attempted cheating (e.g. usage of prohibited additional material, copying from other students, etc.) and unruly behavior will result in a “failed” grade for the exam. Any violation of the rules will be recorded. In the case of cheating or attempted cheating the student may choose to continue the exam even though it will be graded as “failed”. In case of unruly behavior, students will be warned once, and in case of recurrence will not be allowed to finish the exam.

Question	Points	
1		
2.1 *or* 2.2		
3		
4		
5		
6		
7		
8		
9		
Total:		

—— The simulation group wishes you good luck! ——

Questions 1: Continuous Modeling [10 Points]. Predator and Prey.

We have observed the development of three different species with predator-prey relationships and competition in a defined area. We derived a model that considers the following four positive real-valued quantities:

- Number of rabbits R
- Number of hamsters H
- Number of foxes F
- Amount of grass available G

The system is governed by the following interdependencies of these quantities:

- The rabbits multiply steadily, which increases their number proportionally to the current population. The hamster behave analogously.
- The foxes can only multiply when there is food available, meaning either hamsters or rabbits. Their number increases proportionally to the product of the fox population and the rabbit population. The number of foxes also increases proportionally to its product with the hamster population size.
- Additionally, foxes die at a rate proportionally to their current population.
- Foxes eat rabbits and hamsters. The rabbit population decreases proportionally to the product of the number of foxes and the rabbit population size. The same applies to hamsters.
- When there are too many rabbits in the area, the hamsters tend to leave the area. This factor decreases the hamster population proportionally to the square of the number of rabbits.
- Grass grows at a constant rate.
- Both hamsters and rabbits eat grass, diminishing it proportionally to the cumulative size of their populations.
- Rabbits depend on grass as their only food source, starving when there is not enough available. The rabbit population decreases inversely proportional to the amount of grass available. Hamsters can also survive on wheat, and are therefore not affected.

a) [9 Points]
Describe this model as a system of ordinary differential equations. Use symbols a_1, a_2, etc. for **positive** constants.

b) [1 Point]
Mark the one of the following questions that the above continuous model ***cannot*** answer.

1. Can the environment sustain all three prey species in the long term?
2. What is the probability of all foxes dying?
3. Will one of the prey species dominate?

Question 2.1: Semester Assignment „The Sims – Almost Normal Family Life" [20 Points].
IMPORTANT: Answer either Question 2.1 or 2.2, not both! Mark the question you want to have graded!

a) Continuous/Hybrid Behavior [8 Points]
Sketch (graphically) a typical development of the „savings account balance". Briefly **explain why** the system behaves the way you sketched it. In your graph, **mark** and **name** at least three (in total!) different activities and states.

b) Money for damaged school property [6 Points]
In the Semester Assignment, your task was to determine how much money will be spent on damaged school property. **Give** a statistically meaningful answer to this task. **Explain** what that answer means. **Describe** on what basis it has been obtained.

c) Family therapy strategy [6 Points]
Explain in short your strategy of using the available interventions to maximize the probability to stay together for 7 years. **State** the probability that was reached using this strategy.

Question 2.2: Semester Assignment „Star Trek – USS Enterprise in Danger" [20 Points].
IMPORTANT: Answer either Question 2.1 or 2.2, not both! Mark the question you want to have graded!

a) Continuous/Hybrid Behavior [8 Points]
Sketch (graphically) a typical development of the „shield energy level". Briefly **explain why** the system behaves the way you sketched it. In your graph, **mark** and **name** at least three (in total!) different activities and states.

b) Antimatter particle hits [6 Points]
In the Semester Assignment, your task was to determine how many antimatter particles will hit the shield. **Give** a statistically meaningful answer to this task. **Explain** what that answer means. **Describe** on what basis it has been obtained.

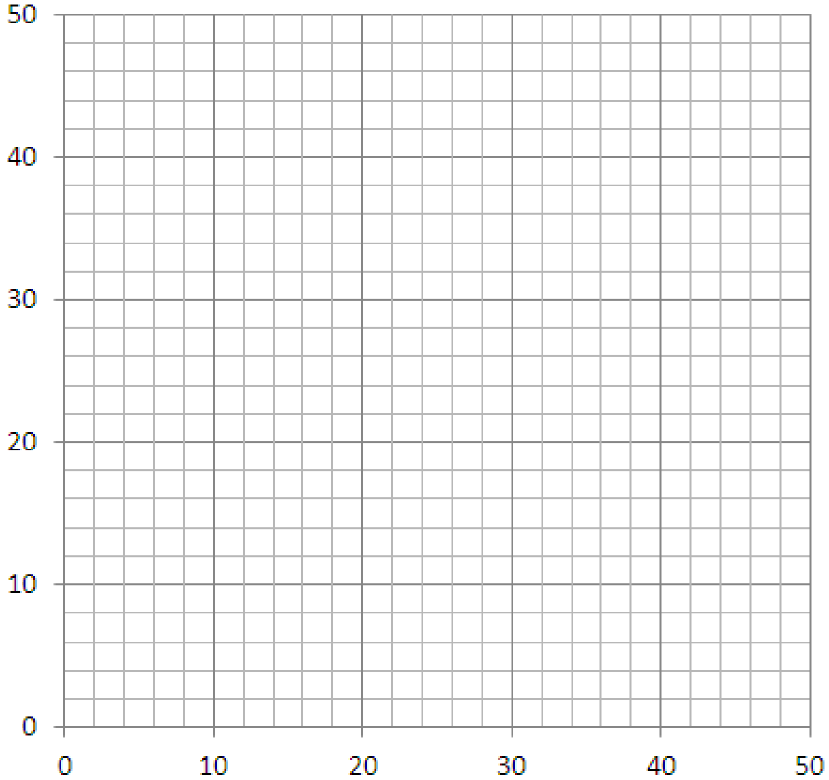
c) Power distribution strategy [6 Points]
Explain in short your strategy for distributing the energy between the engine and shield. **State** the probability that was reached using this strategy.

Question 3: Input Data Analysis [10 Points].

a) Quantile-Quantile-Plot [5 Points]
The following ten numbers were obtained in a measurement:

16.8, 26.9, 33.1, 35.4, 31.0, 38.3, 24.6, 21.7, 43.2, 29.0

You assume that these measurements are distributed according to a Normal(30,4) distribution. To check this assumption, **draw** a Quantile-Quantile-Plot in the empty graph below **and interpret** the result.



b) Chi-Square-Test [5 Points]
You receive a file containing 100 numbers between 0 and 1. These are assigned to ten intervals ("Observed") according to their value. Someone claims that these numbers are uniformly distributed between 0 and 1.

	xMin	***xMax***	***Expected***	***Observed***		
	0	0.1		5		
	0.1	0.2		11		
	0.2	0.3		12		
	0.3	0.4		6		
	0.4	0.5		13		
	0.5	0.6		15		
	0.6	0.7		7		
	0.7	0.8		5		
	0.8	0.9		12		
	0.9	1.0		14		
Sum				100		

What does the Chi-Square-Test say to this hypothesis? Do not merge any classes; round numbers to one decimal place. Use first $\alpha = 0.1$ and then $\alpha = 0.05$. What exactly do these results mean?

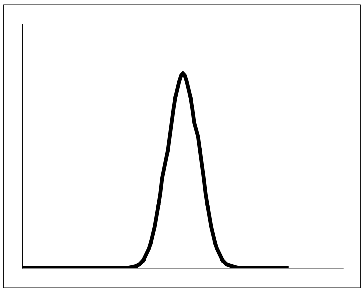
Question 4: Random Variables [10 Points]. Presidential Elections.

a) Probability Density Functions [6 Points]

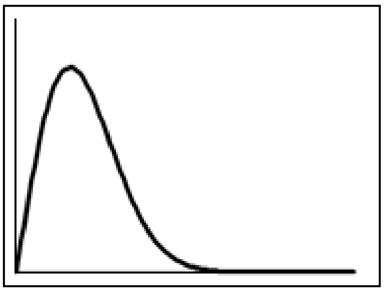
At a polling station, we measured the following random variables:

1. The lifetime of the pencils and pens used for filling out the ballots (voting slips)
2. The time intervals between the arrivals of voters
3. The time one voter spends in the polling booth

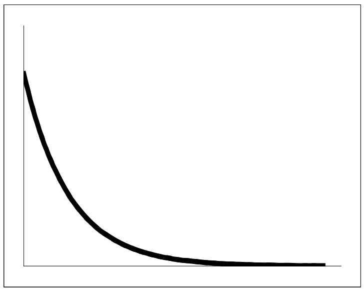
The probability density functions of these distributions are shown here:



A



B



C

Match the graphs A, B and C to the measurements 1, 2 and 3 and **explain** your decision. (Note: Simply naming the corresponding distribution functions is not an explanation for the chosen assignment. The process of elimination is not an explanation either.)

b) Distribution Functions [4 Points]

The voter turnout in presidential elections at a polling station has been found to be distributed according to a triangular distribution with a minimum of 40%, a maximum of 100% and a median of 85%. How many of the 150 polling stations in the District of Columbia can be expected to have a voter turnout between 80% and 90%?

Question 5: Petri Nets [10 Points]. A Maintenance Workshop.

We are looking at a workshop for the maintenance of large diesel engines for locomotives. The engines arrive and are disassembled into their parts. These parts are then refurbished and reassembled. If the following test shows no problems they are finished and leave the workshop. If an engine does not pass the test, it has to go back to assembly to be reworked and checked again.

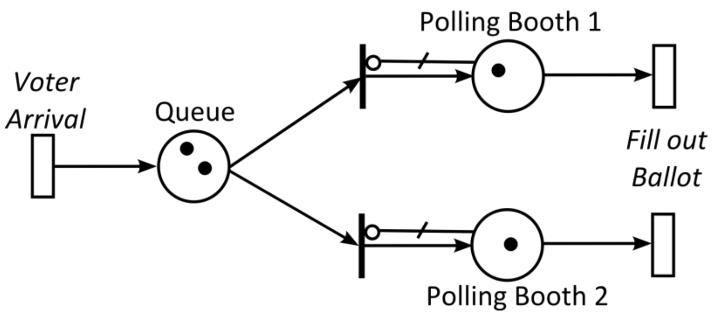
The engines arrive at the workshop singly in random intervals and are stored in a warehouse. The engines are removed from the warehouse and handled one by one. Each engine is disassembled into its five major parts, which takes a randomly distributed amount of time. These parts are stored in a buffer and refurbished one after the other, which again takes a uniformly distributed amount of time for each part. When all five major parts are ready, they are reassembled into one engine, which takes a normally distributed amount of time. After assembly the engines are tested thoroughly. 80% of the engines pass the test and go on to the finishing station, which can process one engine at a time. The finishing takes a random amount of time for each engine and afterwards the engines leave the workshop.
The engines that do not pass the test have to be reworked. This can only be done by the assembly engineers. Reworking an engine has a higher priority than assembling one. Therefore if an engine for rework arrives, the current assembly is interrupted and the rework started. Only after the rework is done, which takes a uniformly distributed amount of time, is the assembly of the other engine resumed where it was left off. The reworked engine needs to be tested again, following the normal process.

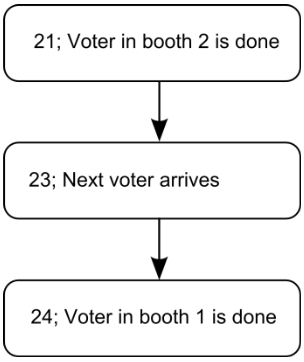
Draw a Petri net model of this system. Assume the following initial state: there are currently no engines awaiting disassembly, three parts waiting for refurbishment and one currently being refurbished. There is an assembly in progress, which has been interrupted for an ongoing rework. Finishing is currently empty.
You can assume that if five parts are assembled, they are the correct five parts.
Mark all transitions that have a race age policy. **List** the transitions that are currently enabled.

Question 6: Progression of a Discrete Simulation [10 Points]. At a Polling Station.

We are modeling the queuing process at a polling station. Voters arrive at random intervals, get their ballot papers. Then they have to enter a polling booth in order to fill out the ballots. People are only allowed to enter the polling booths singly. If both booths are occupied, the voters have to wait until one is empty again. Filling out a ballot takes some time, after which the voters drop the completed ballots into a ballot-box.

The following Petri net represents this process: At time 20, both polling booths are occupied, and there are currently two voters waiting in line. The *Future-Event-List* (FEL) of the system is the following:





The next three inter-arrival times: 4, 2, 7
The next three time intervals for filling out a ballot: 6, 4, 3

a) Simulation progression [7 Points]
In order to sketch the progression of the simulation program from time 20 to time 25, **show** the system state and the next event to occur after *each* state change. **Mark** which events are primary and which are secondary (=conditional).

b) Future Event List [3 Points]
Describe or draw the FEL for time 25, i.e. the FEL after all events for that point in time have been processed.

Question 7: Output Analysis [10 Points]. Next Election Campaign.

The opposition is already pondering over how to win the next presidential election. There are two mutually exclusive ideas for the next electoral campaign: campaign 1 tries to discredit the achievements of the current president and campaign 2 emphasizes the major role of the opposition in some positive achievements of the current government. Since these cannot be pursued both, the campaign staff has to choose one of them.

There exists a complicated simulation model that evaluates the effect of electoral campaigns on the outcome of an election. The campaign staff has used correlated sampling to evaluate the two campaigns, running ten replications each. For every run the resulting percentage of votes for the opposition was noted.

Lap	**Campaign 1**	**Campaign 2**	D_r	$\overline{D}$	$(D_r-\overline{D})^2$		
1	**40**	**38**					
2	**64**	**66**					
3	**39**	**40**					
4	**45**	**47**					
5	**37**	**32**					
6	**59**	**57**					
7	**54**	**50**					
8	**51**	**50**					
9	**50**	**45**					
10	**52**	**56**					

a) Comparison [8 Points]
Statistically **compare** the two sets of replications. Which campaign should be chosen? **Interpret** the results of your calculations and **justify** your decision. (Hints: use empty table cells for your calculations. For the computation of square roots rough estimates are sufficient)

b) Interpretation [2 Points]
Make a statement on the (statistical and practical) significance of the result and **make suggestions** on how to improve the result, if necessary.

Question 8: Discrete Time Markov Chains [10 Points]. Voter Migration

We want to build a simple model of voter migration in presidential elections between the two main parties in the US and nonvoters. We therefore have three possible behaviors at each election:

(1) Voting for the Republicans (2) Voting for the Democrats (3) Not voting at all

We further assume that every potential voter may behave differently at each election, and that the voting decision in the current election only depends on that persons' voting decision of the previous election. Therefore we can assume that voter migration can be represented by a discrete-time Markov chain (DTMC).

We have asked a 1000 people eligible to vote since at least 2006 what their voting decisions were in 2008 and 2012. Assuming that the group is representative we can use this data to build a model of voter migration.

2008 voting decision	2012 voting decision	number		
Republicans	Republicans	150		
Republicans	Democrats	60		
Republicans	nonvoter	90		
Democrats	Republicans	30		
Democrats	Democrats	210		
Democrats	nonvoter	60		
nonvoter	Republicans	120		
nonvoter	Democrats	40		
nonvoter	nonvoter	240		

a) Modelling [4 Points]

Sketch the graphical representation of the discrete-time Markov chain (DTMC) that can be deduced from this statistic.

b) Transient Solution [4 Points]

Assume that a particular voter chose the Republicans in 2008. Using the above model, **compute** the probability that in 2016 he will also vote for the Republicans. (Assume that there is an election every 4 years)

c) *Beyond DTMCs - Hidden Markov Models* [2 Points]

Assume that the outcome of the election and the voting decision directly influence the happiness of the voters. The probability of being happy is 0.8 when the party wins, that they voted for. For nonvoters, the probability to be happy is 0.5, regardless of the wining party.

Consider a person that voted for the Democrats in 2008, **compute** the probability that this particular person is happy now after the 2012 election, which was won by the Democrats,.

Question 9: Miscellaneous [10 Points].

a) Given the *initial value problem* y'(t) = **-** y + 3t with y(0) = -5. This problem is to be solved using the Euler method with step size 1. **Compute** the result at time *t* = 3. [3 Points]

b) **Generate** (pseudo) random numbers that are Triangular(40;100;85) (!) distributed using the *Linear Congruential Method.* What are the first four values x_1 to x_4 generated using the parameters a=9, c=21, m=100 and the seed x_0=22? [3 Points]

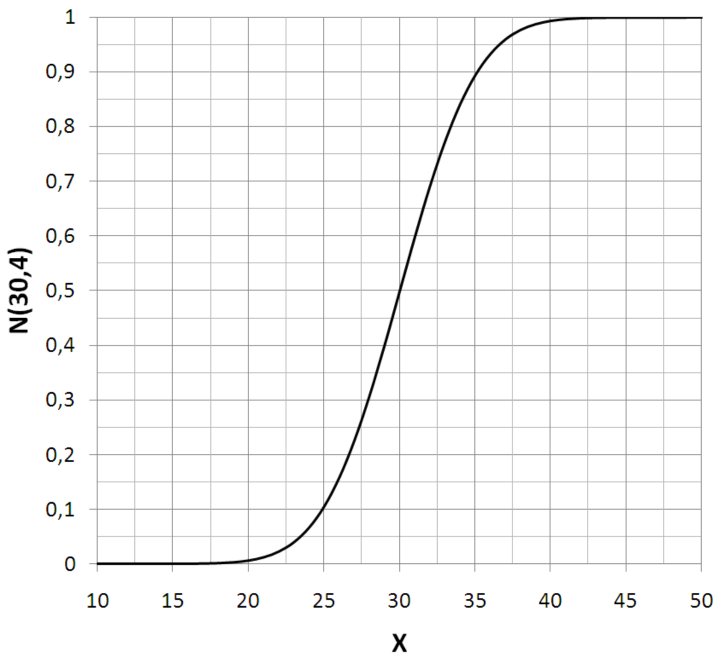
c) We are considering a polling station. **Give** an example for each of the following [3 Points]
(<u>Refer to the definitions from the lecture!</u>)

- an event –
- an activity –
- a delay –
- an entity –
- an attribute –
- a state variable –

d) A queue has formed in front of the polling booths. People arrive there about every 7 minutes and the queue holds on average 2 people. **Compute** how long a person can expect to wait in the queue on average. [1 Point]

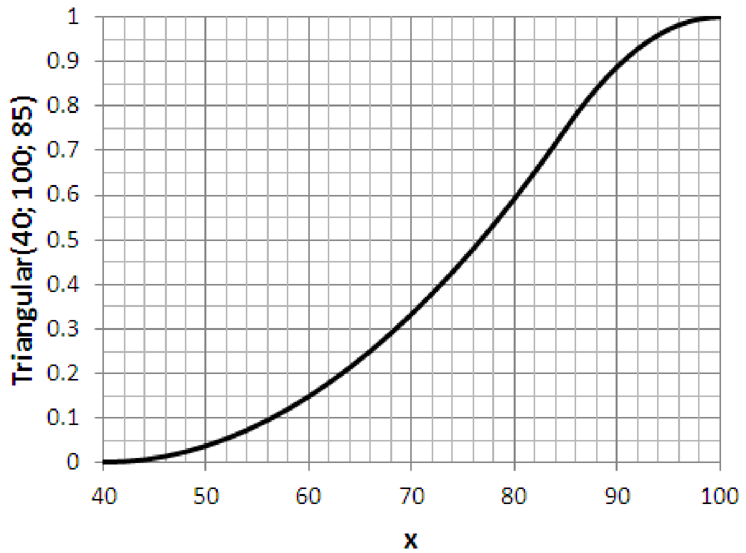
Appendix

Graph of the N(30;4) Distribution



The value of the Student *t*-distribution for 9 degrees of freedom at position 0.05 is 2.26

Triangular(40;100;85) CDF



Some values of the χ^2-Distribution:

		#degrees of freedom					
		7	8	9	10	11	12
α	0.05	14.06	15.51	16.92	18.31	19.68	21.03
	0.10	12.01	13.36	14.68	15.99	17.28	18.55