OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

Simulation Research Group

July 15th, 2014

Exam *Introduction to Simulation*

Total number of points obtainable:	100
Number of questions:	9
Number of pages:	14 (including appendix and empty pages)
Time limit:	120 minutes
Additional material allowed:	Dictionary

Name:			
Student ID#:		Course of studies, year of matriculation	

For your information:

You may answer the questions in either German or English.
Answer all questions according to the contents taught in the lecture.

Rules for written exams at the "Fakultät für Informatik":

Cheating, attempted cheating (e.g. usage of prohibited additional material, copying from other students, etc.) and unruly behavior will result in a "failed" grade for the exam. Any violation of the rules will be recorded. In the case of cheating or attempted cheating the student may choose to continue the exam even though it will be graded as "failed". In case of unruly behavior, students will be warned once, and in case of recurrence will not be allowed to finish the exam.

Question	Points
1	
2.1 *or* 2.2	
3	
4	
5	
6	
7	
8	
9	
Total:	

—— The simulation group wishes you good luck! ——

Questions 1: Continuous Modeling [10 Points]. A Simple Flight Model.

A (strongly simplified) model of the physical properties of a plane in the air, considers the following four positive real-valued quantities:

- Amount of fuel in the tank: f
- Flying altitude (height): h
- Horizontal velocity: v
- Mass of the plane: m

The system is governed by the following interdependencies of these quantities:

- The content of an empty fuel tank does not change.
- The larger the gravitational pull on the plane, the faster the altitude is decreasing. This force is directly proportional to the mass of the plane.
- The horizontal movement of the plane creates a dynamic uplift, counteracting the gravitational pull. The rate of the uplift is proportional to the square of the velocity.
- As long as there is fuel in the tank, it will be burned by the engine. The consumption rate is linearly dependent on the horizontal velocity and at the same time indirectly proportional to the altitude.
- As long as the fuel is burned by the engines, the mass of the plane is decreasing proportional to the fuel consumption.
- The horizontal velocity is constant as long as the engines are getting fuel. Otherwise it is decreasing with a rate proportional to the square of the current velocity, because of the air friction.

a) [10 Points]
Describe this model as a system of ordinary differential equations. Use symbols a_1, a_2, etc. for **positive** constants. **Sketch** the development of altitude and velocity over time.

Question 2.1: Semester Assignment „The Sims – Almost Normal Family Life" [20 Points].
IMPORTANT: Answer either Question 2.1 or 2.2, not both! Mark the question you want to have graded!

a) Continuous/Hybrid Behavior [8 Points]
Sketch (graphically) a typical development of „moms mood". Briefly **explain why** the system behaves the way you sketched it. In your graph, **mark** and **name** at least five (in total!) different activities and states.

b) Duration of Unemployment [6 Points]
In the Semester Assignment, your task was to determine for how long will the father be unemployed. **Give** a statistically meaningful answer to this task. **Explain** what that answer means. **Describe** on what basis it has been obtained.

c) Family therapy strategy [6 Points]
Explain in short your strategy of using the available interventions to maximize the probability to stay together for 7 years. **State** the probability that was reached using this strategy.

Question 2.2: Semester Assignment „Star Trek – USS Enterprise in Danger" [20 Points].
IMPORTANT: Answer either Question 2.1 or 2.2, not both! Mark the question you want to have graded!

a) Continuous/Hybrid Behavior [8 Points]
Sketch (graphically) a typical development of the „shield energy level". Briefly **explain why** the system behaves the way you sketched it. In your graph, **mark** and **name** at least three (in total!) different activities and states.

b) Shield Energy Level [6 Points]
In the Semester Assignment, your task was to determine what the shield energy level will be after 2 hours. **Give** a statistically meaningful answer to this task. **Explain** what that answer means. **Describe** on what basis it has been obtained.

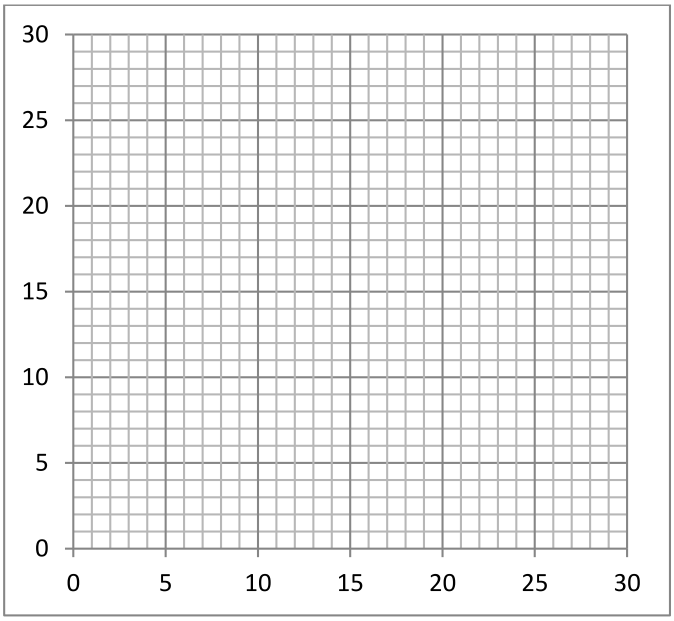
c) Power distribution strategy [6 Points]
Explain in short your strategy for distributing the energy between engines and shields. **State** the survival probability that was reached using this strategy.

Question 3: Input Data Analysis [10 Points].

a) Quantile-Quantile-Plot [5 Points]
The following ten numbers were obtained in a measurement:

16.7, 4.5, 24.3, 2.7, 6.4, 3.6, 5.7, 5.4, 6.2, 12.5

You assume that these measurements are distributed according to a Lognormal(3,0.1) distribution. To check this assumption, **draw** a Quantile-Quantile-Plot in the empty graph below **and interpret** the result.



b) Chi-Square-Test [5 Points]
You receive a file containing 100 numbers between 0 and 1. These are assigned to ten intervals ("Observed") according to their value. Someone claims that these numbers are uniformly distributed between 0 and 1.

	xMin	***xMax***	***Expected***	***Observed***		
	0	0.1		11		
	0.1	0.2		12		
	0.2	0.3		4		
	0.3	0.4		5		
	0.4	0.5		14		
	0.5	0.6		6		
	0.6	0.7		13		
	0.7	0.8		8		
	0.8	0.9		14		
	0.9	1.0		13		
Sum				100		

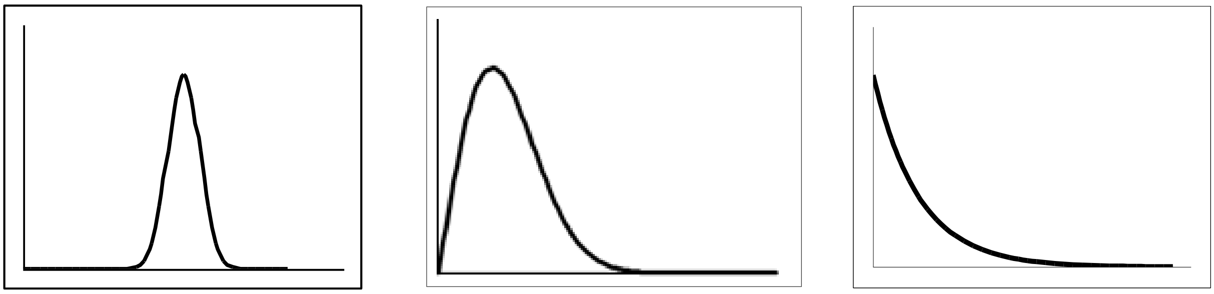
What does the Chi-Square-Test say to this hypothesis? Do not merge any classes; round numbers to one decimal place. Use first $\alpha = 0.1$ and then $\alpha = 0.05$. What exactly do these results mean?

Question 4: Random Variables [10 Points]. At a Gas Station.

a) Probability Density Functions [6 Points]
At a gas station, we measured the following random variables:

1. The lifetime of the gas stations air compressors (“Druckluftpumpe”)
2. The time intervals between the arrivals of walk-in customers
3. The time the cashier needs to serve one customer

The probability density functions of these distributions are shown here:



A B C

Match the graphs A, B and C to the measurements 1, 2 and 3 and **explain** your decision. (Note: Simply naming the corresponding distribution functions is not an explanation for the chosen assignment. The process of elimination is not an explanation either.)

b) Distribution Functions [4 Points]
The time needed for refueling a car has been found to be normally distributed with a mean of 30 seconds and a standard deviation of 4. How many of the 1200 transactions on a given day can be expected to last between 30 and 35 seconds?

Question 5: Petri Nets [10 Points]. Car Sharing.

We are looking at the car pool and rental activities of a car sharing agency. There are two car pools of this particular agency, one in the city and one at the airport nearby. When a car is used, it is either for shopping, or to drive between airport and city. The freeway to and from the airport may be blocked by accidents. Sometimes cars accumulate at the airport, and have to be moved back to the city.

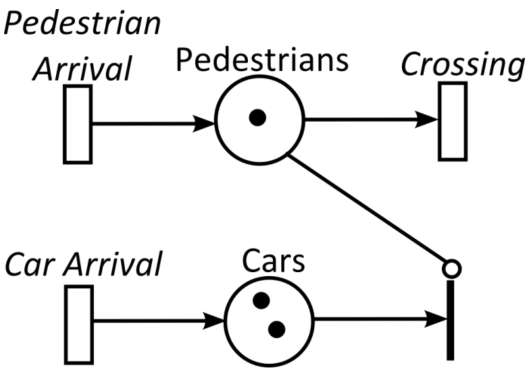
There is a car pool in the city. Requests for these cars arrive at exponentially distributed intervals. When there is no car available, the customer uses another mode of transportation and the request is lost. Otherwise the car is checked out immediately. With a probability of 45% the car will be used for shopping. This takes a randomly distributed amount of time, after which the car is returned to the city car pool. If the customer wants to drive to the airport (55%), he has to take the freeway, which takes a normally distributed amount of time. The request and rental process at the airport car pool is identical to that in the city. If there is no car available, the request is again lost. If there is a car available, after checking out, the customer takes the freeway back to the city, which again takes a normally distributed amount of time.
Sometimes there are accidents on the freeway, that block the traffic in one direction. These happen at exponentially distributed intervals, and it takes a random amount of time to clear the freeway. When there are more than three cars at the airport, and less than three in the city, one car is moved from the airport car pool to the city car pool, which takes a random amount of time.

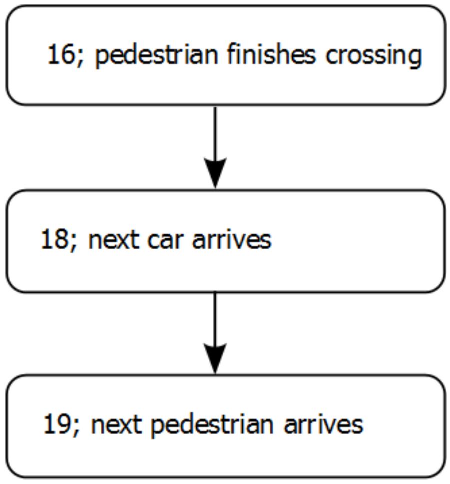
Draw a Petri net model of this system. Assume the following initial state: there are four cars in the city car pool, and two at the airport. One car is on a shopping tour and two are driving back from the airport. The freeway is currently not blocked. One car is currently being transported from airport to city.
Mark all transitions that have a race age policy. **List** the transitions that are currently enabled.

Question 6: Progression of a Discrete Simulation [10 Points]. Pedestrian Crossing.

We are modeling a pedestrian crossing ("Zebrastreifen"). Pedestrians and cars arrive independently at random intervals. Upon arrival, pedestrians start crossing the road immediately, which also takes a random amount of time. If there are currently pedestrians on the road, cars have to wait. Only then can they also leave the scene, which does not take any time.

The following Petri net represents this process: At time 15, there are two cars waiting, and one pedestrian is currently crossing the street. The *Future-Event-List* (FEL) of the system is the following:





The next three inter-arrival times for cars: 8, 7, 5
The next three inter-arrival times for pedestrians: 6, 4, 3
The next three time intervals for crossing the street: 4, 2, 7

a) Simulation progression [7 Points]
In order to sketch the progression of the simulation program from time 15 to time 20, **show** the system state and the next event to occur after *each* state change. **Mark** which events are primary and which are secondary (=conditional).

b) Future Event List [3 Points]
Describe or draw the FEL for time 20, i.e. the FEL after all events for that point in time have been processed.

Question 7: Output Analysis [10 Points]. Traffic Lights or Roundabout.

The City of Magdeburg is currently optimizing the traffic in the city. One activity is to check whether existing traffic lights should be replaced by roundabouts or vice versa. The question is also raised for the intersection of Walther-Rathenau-Straße and Gustav-Adolf-Straße.
Simulation models of the intersection have been built by student teams, one implementing a roundabout, the other a normal traffic lights crossing. The teams have used correlated sampling to evaluate the two layouts. For each run the average waiting time of the cars in seconds has been noted.

Run	Roundabout	Traffic Lights	D_r	$\overline{D}$	$(D_r-\overline{D})^2$		
1	18	19					
2	17	15					
3	6	9					
4	5	5					
5	30	34					
6	11	11					
7	16	23					
8	3	9					
9	20	27					
10	25	29					

a) Comparison [8 Points]
Statistically **compare** the two sets of replications. Which intersection layout should be chosen? **Interpret** the results of your calculations and **justify** your decision. (Hints: use empty table cells for your calculations. For the computation of square roots rough estimates are sufficient)

b) Interpretation [2 Points]
Make a statement on the (statistical and practical) significance of the result and **make suggestions** on how to improve the result, if necessary.

Question 8: Discrete Time Markov Chains [10 Points]. Winter Climate in Germany

We want to build a simple model of the winter climate in Germany. We therefore distinguish between three different classifications of the average winter temperatures:

(1) Too Warm *(2) Too Cold* *(3) Normal*

We assume that the temperature classification of the current winter only depends on the previous winter. Therefore we can assume that the winter climate in Germany can be represented by a discrete-time Markov chain (DTMC).

The following table shows how often each possible combination occurred in the last 100 years. This data can be used to create a model of the winter climate in Germany.

Previous winter	**Following winter**	**number**		
Too Warm	Too Warm	3		
Too Warm	Too Cold	21		
Too Warm	Normal	6		
Too Cold	Too Warm	4		
Too Cold	Too Cold	12		
Too Cold	Normal	24		
Normal	Too Warm	6		
Normal	Too Cold	9		
Normal	Normal	15		

a) Modelling [4 Points]

Sketch the graphical representation of the discrete-time Markov chain (DTMC) that can be deduced from this statistic.

b) Transient Solution [4 Points]

Assume that in 2013 the winter was too cold. Using the above model, **compute** the probability that in 2015 the winter will again be too cold.

c) *Beyond DTMCs - Hidden Markov Models* [2 Points]

Assume that the temperature classification of the winter directly influences the occurrence of snow.
The probability for snow is 0.9 when the winter is too cold, 0.7 when it is normal and 0.4 when it is too warm.
Assuming that the 2013 winter was too cold, **compute** the probability for snow in 2014.

Question 9: Miscellaneous [10 Points].

a) Given the *initial value problem* y'(t) = 2y - 3t with y(0) = 0. This problem is to be solved using the Euler method with step size 1. **Compute** the result at time t = 3. [3 Points]

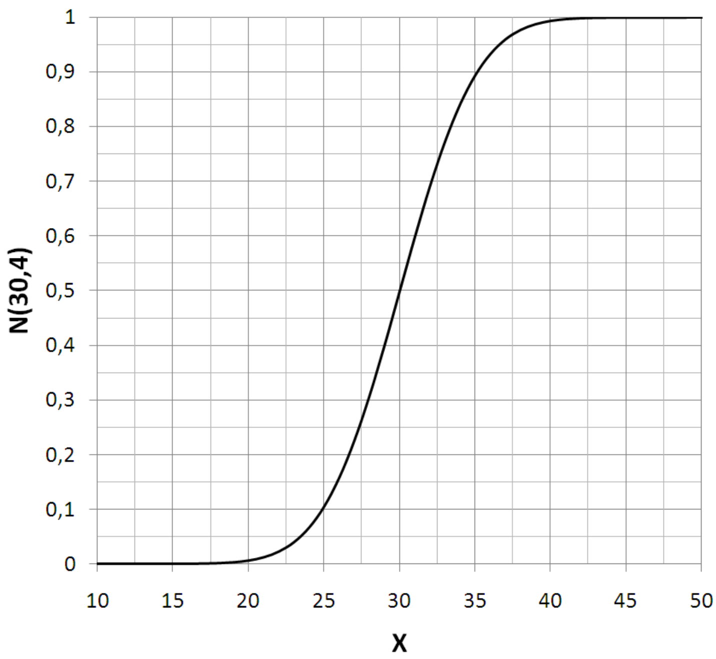
b) **Generate** (pseudo) random numbers that are Lognormal(3;0.1) (!) distributed using the *Linear Congruential Method.* What are the first four values x_1 to x_4 generated using the parameters a=3, c=62, m=100 and the seed x_0=31? [4 Points]

c) We are considering a gas station. **Give** an example for each of the following [3 Points]
(Refer to the definitions from the lecture!)

- an event –
- an activity –
- a delay –
- an entity –
- an attribute –
- a state variable –

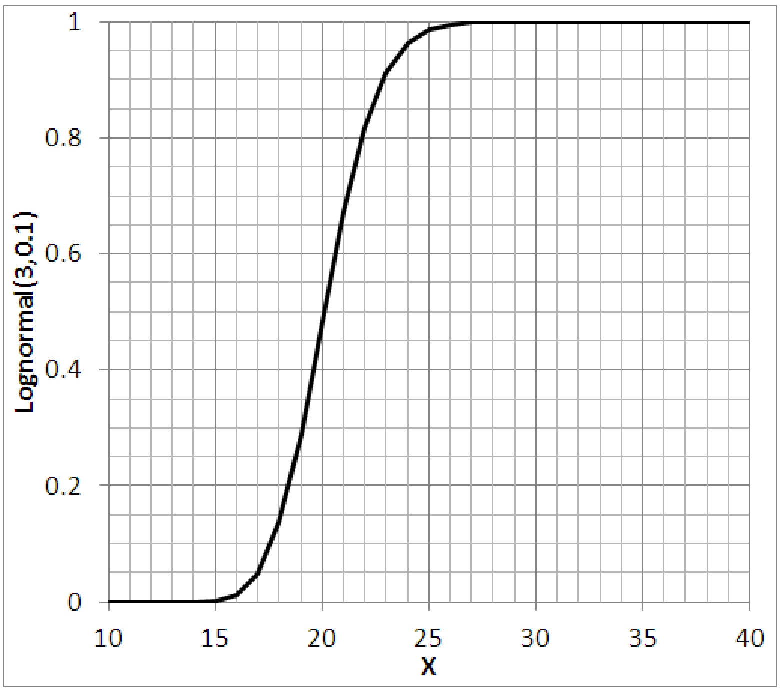
Appendix

Graph of the N(30;4) Distribution



The value of the Student *t*-distribution for 9 degrees of freedom at position 0.05 is 2.26

Lognormal(3;0.1) CDF



Some values of the χ^2-Distribution:

		#degrees of freedom					
		7	8	9	10	11	12
α	0.05	14.06	15.51	16.92	18.31	19.68	21.03
	0.10	12.01	13.36	14.68	15.99	17.28	18.55