OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG | INF | FAKULTÄT FÜR INFORMATIK

Lehrstuhl für Simulation

11. Februar 2010

Written Examination (Klausur) *Introduction to Simulation*

Total number of points obtainable:	90
Number of questions:	8
Number of pages:	12 (including appendix and empty pages)
Time limit:	120 minutes
Additional material allowed:	none

Name:			
Matrikelnummer:		Studiengang / Matrikeljahr:	

For your information:

Die Antworten können auch in englischer Sprache erfolgen.
You may answer the questions in German or English.

Rules for written exams at the "Fakultät für Informatik":

Cheating, attempted cheating (e.g. usage of prohibited additional material, copying from other students) and unruly behavior will result in a "failed" grade for the exam. Any violation of the rules will be recorded. In the case of cheating or attempted cheating the student may choose to continue the exam even though it will be graded as "failed". In case of unruly behavior, students will be warned once, and in case of recurrence will not be allowed to finish the exam.

Question	Points	
1		
2		
3		
4		
5		
6		
7		
8		
Total:		

—— The simulation group wishes you good luck! ——

Question 1: Continuous modeling [10 Points]. Epidemiology.

The spread of an infectious disease in a closed population will be analyzed using system dynamics. The so-called SEICR model of this situation contains the following groups of people, in the form of positive real-valued quantities:

- Susceptible individuals: S
- Exposed, but not yet infectious individuals: E
- Infectious individuals: I
- Currently non-infectious carriers: C
- Recovered and immune individuals: R

The following assumptions are made regarding the interdependencies of these population groups:

- The infection of susceptible individuals happens proportional to their number and to the proportion of infectious individuals in the whole population.
- Newly infected individuals are not infectious initially, but only after an exponentially distributed incubation period. Therefore the number of people in the incubation phase is reduced proportional to itself.
- After incubation, an exposed individual turns into an actually infectious individual.
- The number of infectious individuals decreases proportionally to itself through fading of the symptoms.
- After recovery, only a certain percentage p of the no longer infected is actually healed. The rest becomes a currently not infectious carrier of the disease.
- Among these not infectious carriers the disease can break out again with a constant rate and turns them into infectious individuals again. Therefore the number of carriers decreases proportionally to itself.
- Recovered individuals can lose their immunity and therefore change back into group S, which decreases the group of recovered individuals proportionally to itself.

Hint: In this model's closed population individuals may neither disappear nor suddenly appear.

a) [9 Points]
Describe this model in form of a system of ordinary differential equations! Use symbols a_1, a_2, etc. for positive constants.

b) [1 Point]
Which of the following questions can<u>not</u> be answered by such a model?

1. What is the probability that the disease is eradicated in the population?
2. How many individuals will be infected by the disease presumably?
3. How long will the disease persist in the population?

Question 2: Semester Assignment „Back to the Future" [20 Points].

a) Continuous behavior [10 Points]
Sketch a typical development of the „LEVEL OF AFFECTION" for the case that Lorraine and George fall in love at the end of the night. Mark and name at least five (in total!) different activities and states! Briefly explain the behavior!

b) AnyLogic-Modeling [5 Points]
Explain how and with the use of which AnyLogic model elements you modeled the following:
„Doc Brown takes one step upwards" (also consider the slipping down!)

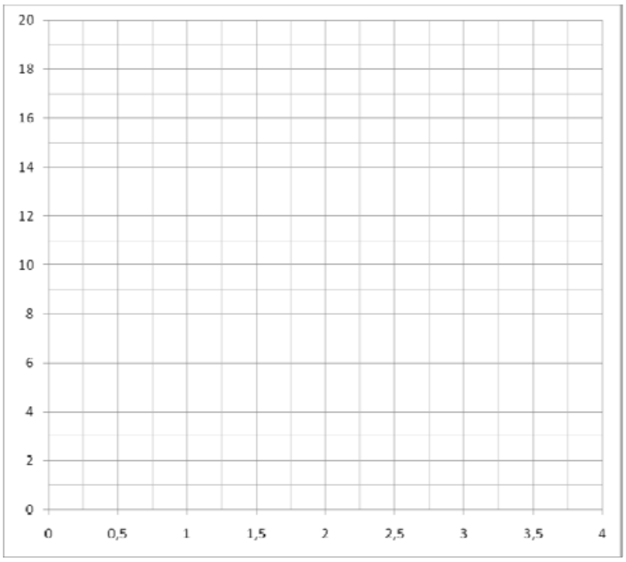
c) George und Biff [5 Points]
How often during one dance night does George McFly beat up Biff Tannen? What is a statistically meaningful answer? What does this mean and on what basis has it been obtained?

Question 3: Input data analysis [10 Points].

a) Quantile-Quantile-Plot [5 Points]
The following ten numbers were obtained in a measurement:

7.9, 17.3, 14.9, 4.5, 10.1, 6.5, 13.4, 12.2, 9.1, 11.1

You assume that these measurements are distributed according to a Weibull distribution. To check this assumption, draw a Quantile-Quantile-Plot in the empty graph below and interpret the result!



b) Chi-Square-Test [5 Points]
You receive a file containing 200 numbers between 0 and 1. These are assigned to ten intervals ("Observed") according to their value. Someone claims that these numbers were produced by a random number generator. <u>(In this case the number of free distribution parameters s=0.)</u>

	xMin	***xMax***	***Expected***	***Observed***		
	0	0.1		11		
	0.1	0.2		17		
	0.2	0.3		26		
	0.3	0.4		25		
	0.4	0.5		15		
	0.5	0.6		22		
	0.6	0.7		12		
	0.7	0.8		24		
	0.8	0.9		21		
	0.9	1.0		27		
Sum				200		

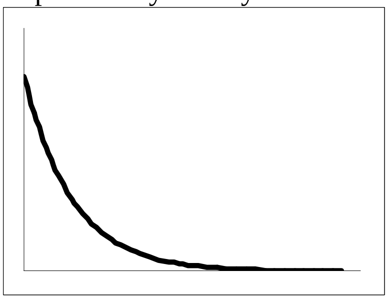
What does the Chi-Square-Test say? Do not merge any classes; round numbers to one decimal place. Use first $\alpha = 0.1$ and then $\alpha = 0.05$. What exactly do these results mean?

Question 4: Random variables [10 Points]. Renewable energies.

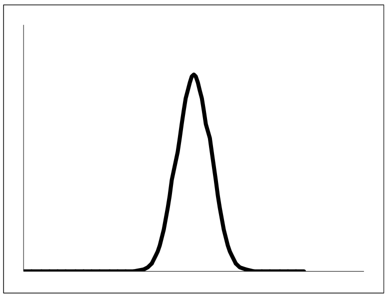
a) Density functions [6 Points]
In the context of constructing an offshore wind farm the company Nordwind has determined the following random variables:

1. Duration of a transport from the last reloading point to the harbor
2. Inter-arrival times of other trucks at the harbor
3. Life expectancy of a wind turbine in offshore service

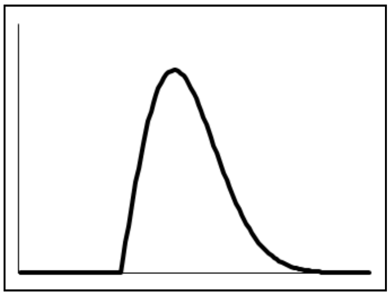
The probability density functions of these distributions are shown here:



A



B



C

Match the graphs A, B and C to the measurements 1, 2 and 3 and explain your decision! (Note: Simply naming the corresponding distribution functions is not an explanation for the chosen assignment!)

b) Exponential distribution [2 Points]
In 2009 the EU decided to prohibit ordinary incandescent light bulbs. Therefore the university needs to rethink the kind of lamps they will buy in the future. Our university needs mostly energy-saving lamps of one type, which has an exponentially distributed lifetime of 50,000 hours. Because of the higher prices, one may also buy used, but still functional energy-saving lamps on the market. The university has two options now: new lamps for 2.52 € a piece, or lamps that have been in use for 25,000 hours for 1.26 €. Which lamps do you recommend and why?

c) Distribution functions [2 Points]
The degree of efficiency of the solar cells produced by the company PhotoEnergy is distributed according to N(30;4). How many of the 60,000 cells produced per day have a degree of efficiency between 30% and 35%?

Question 5: Petri nets [10 Points]. A medical practice.

Each day, Doctor Cancer, M.D. treats many patients in her orthopedic practice, some with and some without an appointment. Furthermore, telephone calls arrive and interfere with the regular workflow. The complete workflow seen from the patients' perspective is described below:

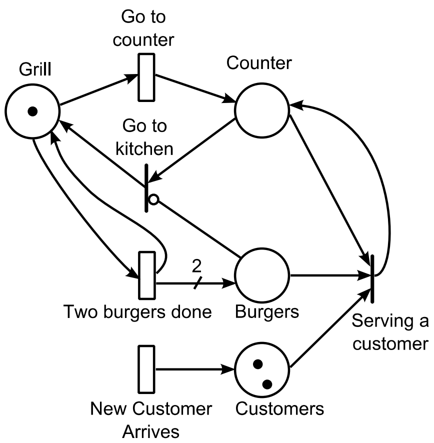
Patients with an appointment arrive in normally-distributed intervals. With probability p they need to get x-rayed before they can continue to the waiting room. X-raying a patient takes a fixed amount of time. Patient that do not need an x-ray, go directly to the waiting room.
Additionally, patients without appointments arrive completely independent of each other (they are not emergencies). The nurses are responsible for the waiting room and do not want it to be too full. Therefore, a patient without an appointment may only go to the waiting room, if there are no more than 17 people waiting, including the ones currently being x-rayed. Otherwise the patient has to leave and look for another doctor.
There is only one treatment room, where Doctor Cancer treats patients. Therefore, she can only treat one patient at a time. The treatment is finished after a uniformly-distributed time period.
When a telephone call arrives, the current treatment is interrupted, and afterwards continued at the same point. Telephone calls arrive in exponentially distributed intervals and have a normally distributed duration.

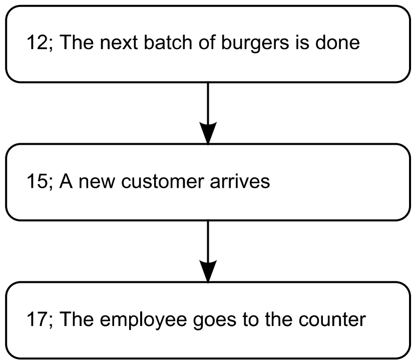
Draw a Petri net model of this system! Assume the following initial state: There are currently two patients in the waiting room and one being x-rayed. One patient is currently being treated by Doctor Cancer. Mark all transitions that have a race age policy! Which transitions are currently enabled?

Question 6: Progression of a discrete simulation [10 Points]. In a fast food restaurant.

„Donald's Fried Burger“ is a rigorously organized fast food restaurant, which can be handled by a single employee. A computer tells him, how long he has to fry burgers (always two at a time). When some burgers are done, they are stored for further use and the employee starts frying another batch of two. When the time determined by the computer is over, the employee throws the half-done burgers away and moves to the counter. Customers can arrive at any point in time and patiently queue up to wait in front of the counter. They are served immediately, if the employee is at the counter and there are burgers available. Every customers receives exactly one burger and then immediately leaves the restaurant. As soon as there are no more burgers left, the employee returns to the kitchen to fry some more.

The following Petri net represents this system. At time 11 no burgers are done, and the employee is currently frying burgers in the kitchen. Two customers are already waiting in front of the counter. The *Future-Event-List* (FEL) at time 11 is the following:





The next three time intervals for frying two burgers are: 6, 3 and 3.
The next three computer-prescribed time periods to stay in the kitchen are: 7, 4 and 11.
The next three inter-arrival times of customers are: 4, 3 and 5.

a) Simulation progression [7 Points]
Sketch the progression of the simulation program from time 11 to time 21. Indicate the changes in the system state and which events are primary and secondary.

b) Future Event List [3 Points]
What is the content of the FEL at time 21?

Question 7: Output analysis [10 Points]. In a dental practice.

The dental practice of Doctor Crown, DDS, is planning to move. Two properties are short-listed, in one of them, two treatment rooms could be set up, in the other one only one. Two treatment rooms would enable to handle more patient, since the nurses could prepare the patients in parallel with the actual treatments. However, the larger practice also results in more costs. You are to decide now, which of the practice settings would be more profitable.

Both configurations are simulated with 10 runs each, with all 20 runs being statistically independent of each other. This results in the following samples for profit per hour for each system:

Run Nr.	1 Room	2 Rooms	D_r	$\overline{D}$	$(D_r-\overline{D})^2$		
1	31	30					
2	33	35					
3	56	54					
4	31	27					
5	37	41					
6	51	49					
7	41	42					
8	33	35					
9	45	40					
10	49	44					

a) Comparison [6 Points]
Which property should be rented? (Hints: Use the empty table cells for your calculations! Rough estimates for square roots are sufficient.)

b) Improving the result [4 Points]
Name two alternative ways to improve this result. Explain your suggestions!

Question 8: Miscellaneous [10 Points].

a) Consider the following initial value problem $y' = y - 2t$, $y(0) = 3$. Solve it using Euler's method with a time step size of 1. What value do you get for time $t = 3$? [3 Points]

b) We want to generate (pseudo) random numbers, which are N(30;4) (!) distributed. Use the Linear Congruential Method. What are the first four values (roughly) that are obtained when choosing the parameters: $a = 5$, $c = 17$, $m = 100$ and the *seed* $x_0 = 11$? [3 Points]

c) We are considering an exercise class in our faculty. Give one example for each of the following [2 Points] (Refer to the definitions given in the lecture!)

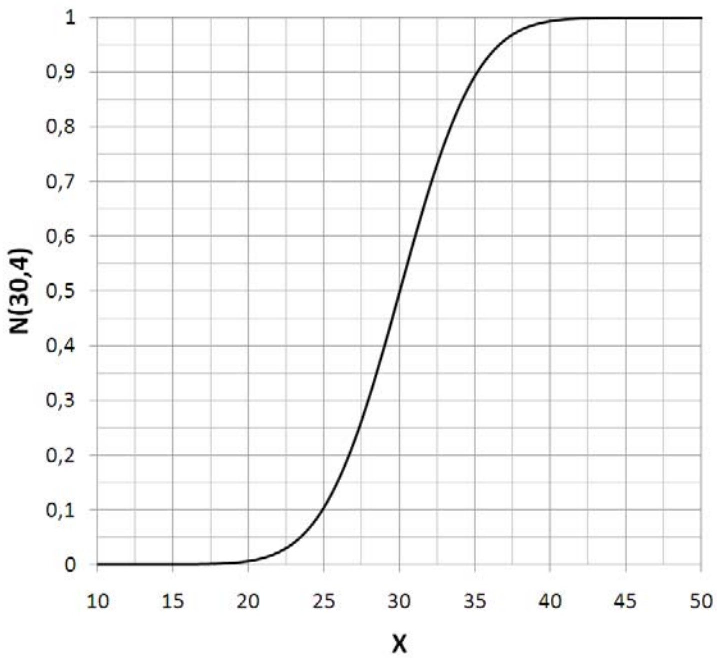
- an event –
- an activity –
- a delay –
- an entity –
- an attribute –

d) What happens to the global error of Euler's method when the step size is quadrupled? [1 Point]

e) A queue has formed in front of the examination office. Students arrive there about every five minutes and the queue holds on average two people. How long should a student expect to wait in the queue? [1 Point]

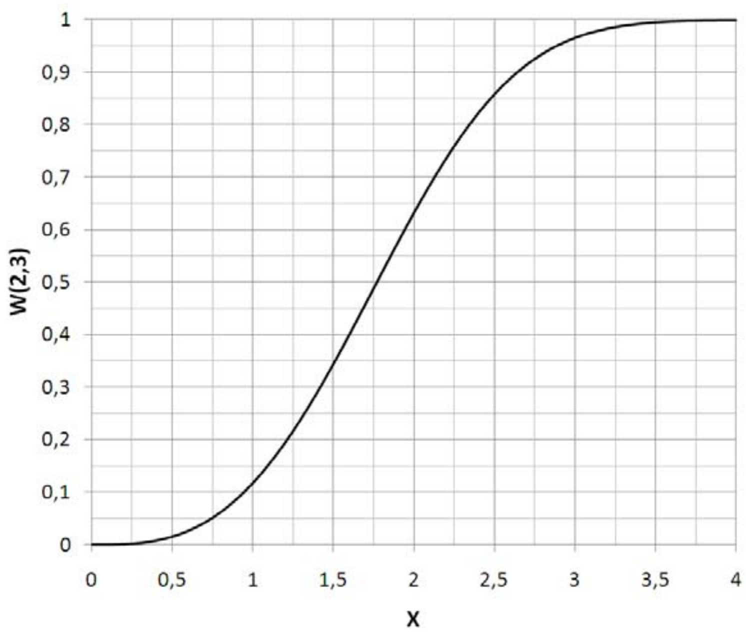
Appendix

Graph of the N(30;4) distribution



The value of the Student *t*-distribution for $\alpha = 0.05$ and 9 degrees of freedom is 2.26

Graph of the W(2;3) distribution



Some values of the χ^2-distribution:

		Degrees of freedom				
		8	9	10	11	12
α	0.05	15.51	16.92	18.31	19.68	21.03
	0.1	13.36	14.68	15.99	17.28	18.55