# Cheat Sheet: Mathematical induction

Task Definition: Show that the Equation $\sum_{i=0}^{n} f(i) = g(n)$ is true.

## 1st Step: Title

- write down what needs to be shown
- "to Show: $\sum_{i=0}^{n} f(i) = g(n)$ for the natural numbers $\mathbb{N}$"

## 2nd Step: Induction Start

- Insert the smallest element, insert $n = 0$ or $n = 1$
- calculate and check whether the equation applies
- 'I.Sta.: $\sum_{i=0}^{0} f(i) = c = g(0)$ true statement.'

## 3rd Step: Induction Step

### 3.1. Induction Assumption

- Write down that the equation already applies up to $n$
- 'I.A.: $\sum_{i=0}^{n} f(i) = g(n)$ is valid for an arbitrary but fixed $n \in \mathbb{N}$.'

### 3.2 Induction Claim

- insert $n + 1$ everywhere in the equation $n$
- 'I.Cl.:' $\sum_{i=0}^{n+1} f(i) = g(n+1)$
- if possible, simplify or multiply out the statement.

### 3.3 Induction Proof

- Prove that the Claim follows from the Assumption (I.A. $\rightarrow$ I.Cl.)

- To do this, break down into the sum $\sum_{i=0}^{n} f(i)$ and the last summand f(n+1)

$$\sum_{i=0}^{n+1} f(i) = \underbrace{f(0) + f(1) + \ldots + f(n)}_{\text{sum to n}} + f(n+1) = \sum_{i=0}^{n} f(i) + f(n+1)$$

- then replace the front sum via I.A. $\sum_{i=0}^{n} f(i) = g(n)$

$$= \underbrace{\sum_{i=0}^{n} f(i)} + f(n+1) = \underbrace{g(n)} + f(n+1)$$

- Finally, transform the terms until $g(n+1)$ comes out

$$= g(n) + f(n+1) = \ldots = g(n+1)$$

↳ $\sum_{i=0}^{n} f(i) = g(n)$ applies. ■