



Magdeburg, den 13.09.2024

Aufgabe 1 : *Fundamental Algorithms*

a) *The following code implements a recursive version of the binary search algorithm.*

```
1 def find(a: list[int], x: int) -> int | None:
2     def _find(a: list[int], x: int, l: int, r: int) -> int | None:
3         if l > r:
4             return None
5         m: int = (l+r) // 2
6         if a[m] == x:
7             return m
8
9         if x < a[m]:
10            return _find(a, x, l, m - 1)
11        else:
12            return _find(a, x, m + 1, r)
13
14    return _find(a, x, 0, len(a)-1)
```

- *What is the meaning of the return value of `find`?*
- *Define and implement an iterative function `ifind` that implements binary search without using recursion. Do not use any libraries or built-in functions that would solve the problem directly.*
- *Assume that you are allowed to make only 11 comparisons. What is the size of the largest array that you can search and guarantee that you return the correct answer?
(**Hint:** To make things a bit easier, you should only consider the comparison in line 9 as comparison in the context of this question.)*

[5 points]



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b) Given the following function, explain what each function calculates and what its worst case runtime complexity is (Big-O notation).

```
1 def a(n: int) -> int:
2     if n <= 1:
3         return 1
4     else:
5         return n * a(n-1)
6
7
8 def b(n: int) -> int:
9     if n <= 0:
10        return 0
11    else:
12        return 1 + b(n//2)
13
14 def c(n: int) -> int:
15     if n <= 0:
16         return 0
17     elif n <= 2:
18         return 1
19     else:
20         return c(n-2) + c(n-1)
```

[6 points]

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Aufgabe 2 : Object Oriented Programming

a) Consider the following class hierarchy, which output is generated by the lines 30, 33, 34, and 37?

```
1 class Pokemon:
2     def attack(self, s: str) -> None:
3         raise NotImplementedError
4
5     def move(self, target: str) -> None:
6         print(f"Teleporting to {target}.")
7
8 class Eevee(Pokemon):
9     def attack(self, s: str = "") -> None:
10        print("Tackle!")
11
12    def move(self, target: str) -> None:
13        print(f"Run away from {target}.")
14
15 class Vaporeon(Eevee):
16    def attack(self, target: str) -> None:
17        super().move(target)
18        print("Water Gun!")
19        self.attack(target)
20
21 class Magikarp(Pokemon):
22    def attack(self, s: str) -> None:
23        super().attack(s)
24
25    def move(self, target: str) -> None:
26        print(f"Flop around.")
27
28 if __name__ == "__main__":
29     garados: Magikarp = Magikarp()
30     garados.move("Kanto")
31
32     jolteon: Eevee = Eevee()
33     jolteon.attack()
34     jolteon.move("Alola")
35
36     flareon: Pokemon = Vaporeon()
37     flareon.attack("Pikachu")
```

[4 points]

	Output
line 30:	
line 33:	
line 34:	
line 37:	

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Aufgabe 3 : Lists

a)

```
1 class DoubleLinkedListNode:
2     def __init__(self, item=None, prev=None, next=None):
3         self._item = item
4         self._prev = prev
5         self._next = next
6
7
8 class DoubleLinkedList:
9     def __init__(self):
10        self._head = DoubleLinkedListNode() # an empty DLL always
11        self._tail = DoubleLinkedListNode(prev=self._head) # contains at least
12        self._head._next = self._tail # two empty nodes
13
14        def insert_before(self, node: DoubleLinkedListNode):
15            pass
16
17        def remove(self, node: DoubleLinkedListNode):
18            pass
```

Implement the two missing functions `insert_before` and `remove`. You may assume that all argumentes are sufficiently instantiated (i.e., not `None`) and – in the case of `remove` – are contained in the double linked list. You can also directly access all members of the `DoubleLinkedListNode` class without using getter or setter functions. [4 points]



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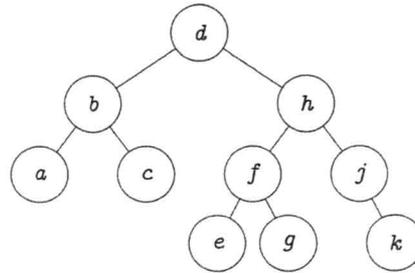
b) *The implementation of the doubly linked list from the lecture slides and the previous task use separate dummy nodes for the `_head` and `_tail` pointers. Why? What would change, if we just referenced the first and last element directly? [2 points]*



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Aufgabe 4 : *Tree Structures*

a) Consider the following binary search tree:



- Delete – in that order – the nodes **a**, **j**, and **d**. Draw the resulting tree after every removal.

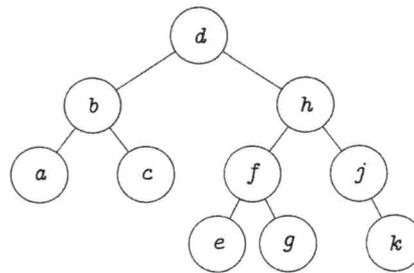
[4 points]



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b) Given the following binary tree, what is the output of

- a preorder traversal,
- an inorder traversal,
- a postorder traversal,
- a levelorder traversal?



preorder traversal:	
inorder traversal:	
postorder traversal:	
levelorder traversal:	

[4 points]



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Aufgabe 5 : *Balanced Trees*

a)

- Explain briefly what an AVL tree is.
- Insert the following sequence of numbers into an AVL tree. Sketch the tree after every insertion and before/after every reorganisation step.

[5, 2, 1, 6, 3, 4, 7, 0]

[5 points]



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b)

- Explain **briefly** what an 2-3-4 tree is.
- Insert the following sequence of numbers into an 2-3-4 tree. Sketch the tree after every insertion and before/after every reorganisation step.

[50, 20, 60, 10, 8, 15, 32, 26, 48, 42]

[5 points]

Aufgabe 6 : Hashs

a) Insert the following sequence of numbers into a hash table \mathcal{H} .

[15, 9, 1, 10, 8, 3]

The hash table has the size $m = 7$. Consider the following two hash functions:

$$h_1(x) = 2x - 1$$

$$h_2(x) = 3x + 1.$$

x	1	3	8	9	10	15
$h_1(x)$						
$h_2(x)$						

1. Use open addressing with linear probing for collision handling. Use h_1 as hash function! Additionally count the number of collisions.

i	0	1	2	3	4	5	6
$\mathcal{H}[i]$							

2. Use open addressing with double hashing for collision handling. Use h_1 and h_2 as hash functions! Additionally count the number of collisions.

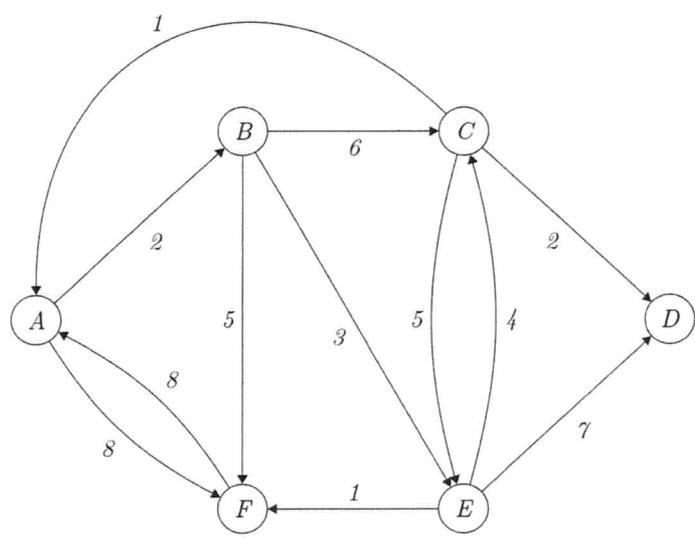
i	0	1	2	3	4	5	6
$\mathcal{H}[i]$							

3. What advantage does double hashing have in general over linear probing?
 4. To avoid having too many collisions we might consider a randomized hash function. Why is this a bad idea?

[9 points]

Aufgabe 7 : Graphs

a) Apply Dijkstra's Algorithm on the following graph, starting from node A.

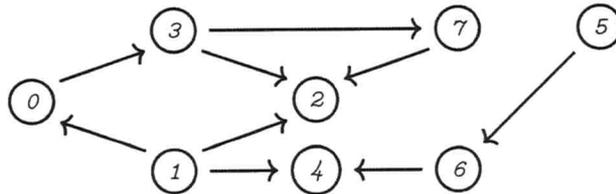


	A	B	C	D	E	F
<i>parent</i>	-					
<i>dist. to A</i>	0					

[4 points]

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b) You are given the following graph:



Calculate a topological ordering of this graph. If at any stage more than one node is available, choose the one with the lower number.

[3 points]

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Aufgabe 8 : True or False

a) Decide whether the following statements are true or false. You do not need to justify your answer. Every correct answer gives 0.5 points, every wrong answer gives -0.5 points. Not answering a question gives neither a malus nor a bonus. You will score at least 0 points on this task.

1. $\log_3 2^n \in \mathcal{O}(n)$
2. Bubble sort can be faster than insertion sort in the best case.
3. The fastest way to build a heap from an array is in $\mathcal{O}(n)$.
4. The B in B-Tree stands for binary.
5. Merge sort can be used as an external sorting algorithm.
6. Python programs are compiled before they are executed.
7. In Python classes can inherit from more than one parent class.
8. Prefixing a class member with `__` means that it cannot be accessed from outside of the defining class.
9. In a binary min heap the largest element can be found in one of the leaves.
10. Solving the knapsack problem with dynamic programming requires $\mathcal{O}(n^C)$ steps.

	true	false
1	<input type="checkbox"/>	<input type="checkbox"/>
2	<input type="checkbox"/>	<input type="checkbox"/>
3	<input type="checkbox"/>	<input type="checkbox"/>
4	<input type="checkbox"/>	<input type="checkbox"/>
5	<input type="checkbox"/>	<input type="checkbox"/>
6	<input type="checkbox"/>	<input type="checkbox"/>
7	<input type="checkbox"/>	<input type="checkbox"/>
8	<input type="checkbox"/>	<input type="checkbox"/>
9	<input type="checkbox"/>	<input type="checkbox"/>
10	<input type="checkbox"/>	<input type="checkbox"/>

[5 points]